

Generalizability in Causal Inference

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Outline

1. What is causal inference?
2. Observational causal inference (internal validity)
3. Transportability of causal effects
4. Recovering from selection bias
5. Data fusion

What is causal inference?

Causal assumptions → Causal conclusions

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To make the leap from _____ to _____ we need **a model**. *The model allows us to go from assumptions to conclusions, and the assumptions of your model must be in the same level of the leap you want to make.*

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NEW RESEARCH IN

Physical Sciences

Causal inference and the data-fusion problem

Elias Bareinboim and Judea Pearl

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Outputs: We will go over each of these for TR and SB problems. But first a quick review.

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Observational Causal Inference

Observational Distribution → Experimental Distribution

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Structural models: combine the power of potential outcomes, structural equations, and graphs.

The structural model

The **structural model is our oracle**. With a fully specified structural model we can answer **any** causal or counterfactual question.

Functional assignments

$$\begin{aligned}M : \quad Z &= f_z(U_z) \\ X &= f_x(Z, U_x) \\ Y &= f_y(X, Z, U_y)\end{aligned}$$

Distribution unobserved factors

$$P : \quad P(U_z, U_x, U_y)$$

Causal (and counterfactual) quantities are **defined in terms of our model**.

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In most cases we don't have a fully specified model, but only a partial understanding of what is going on. How can we encode that knowledge?

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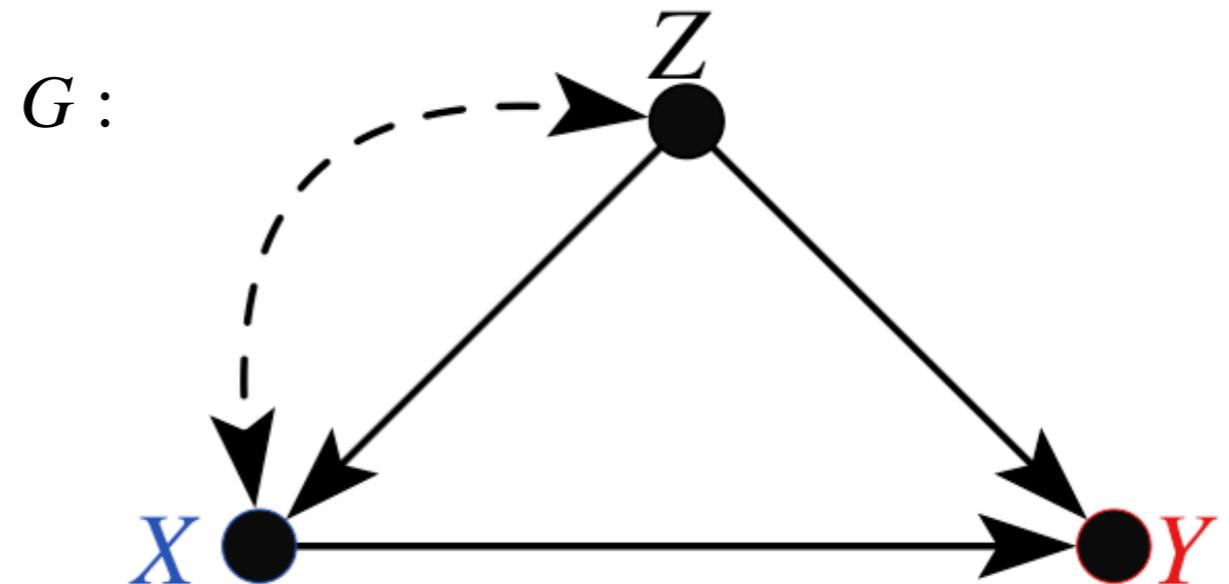
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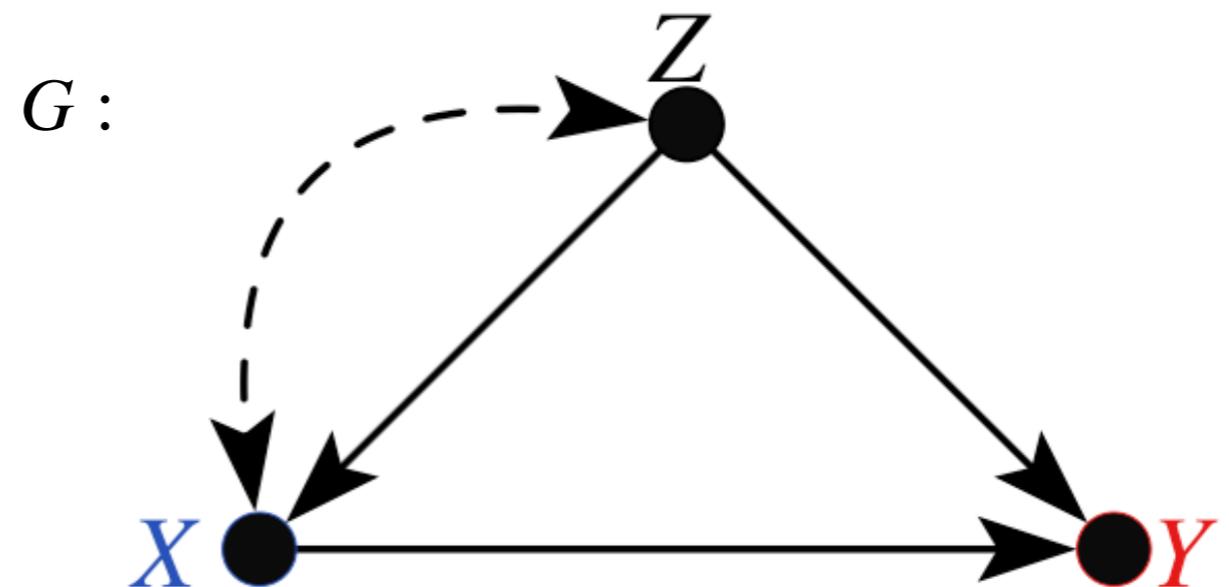
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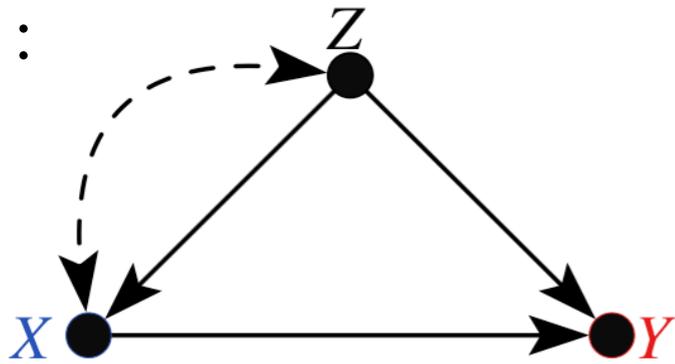


The question of whether our partial understanding + the data we have is sufficient for answering our query is known as the **identification problem**.

The identification problem

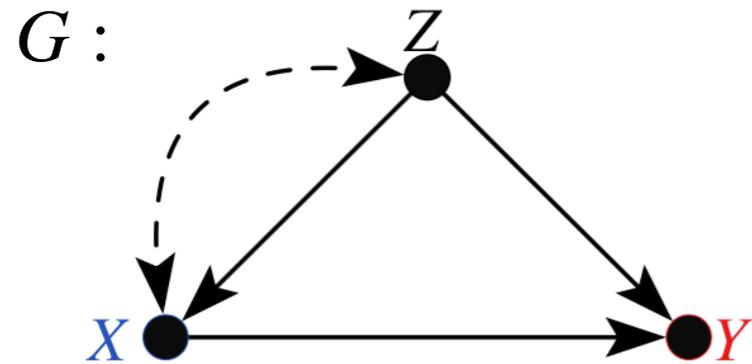
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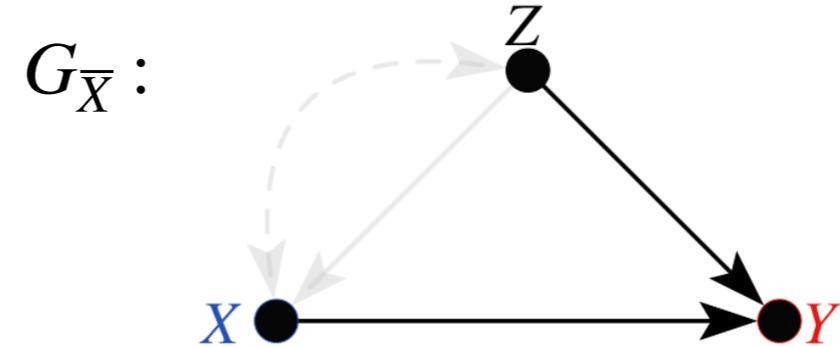


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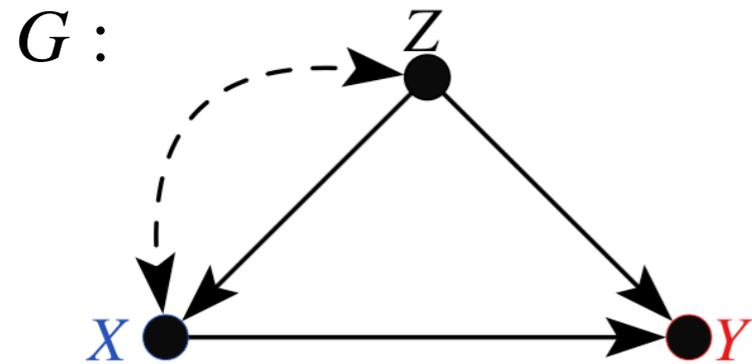


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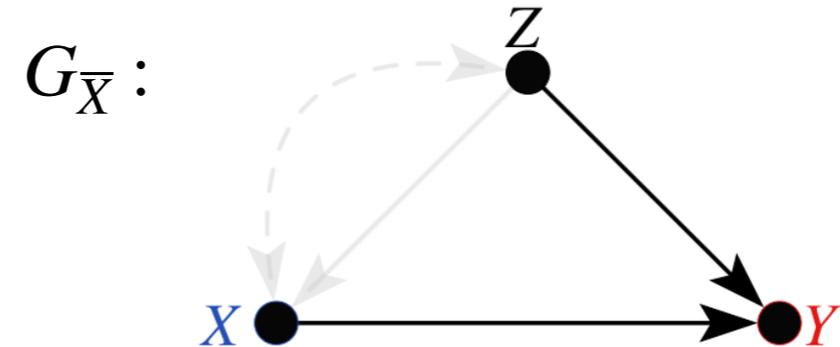


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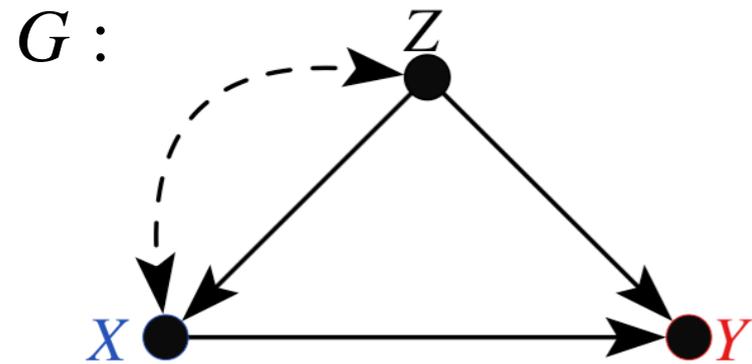
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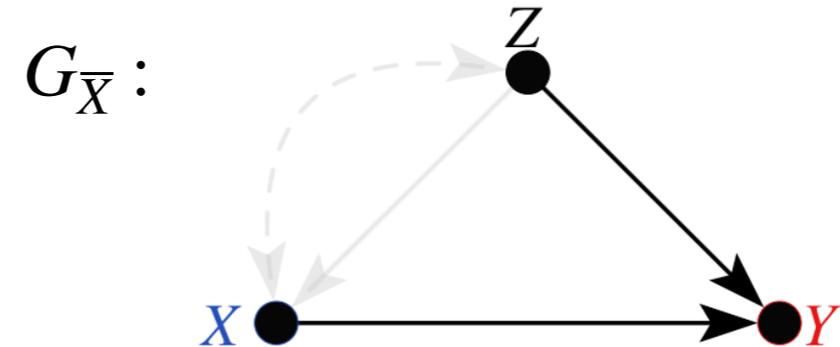
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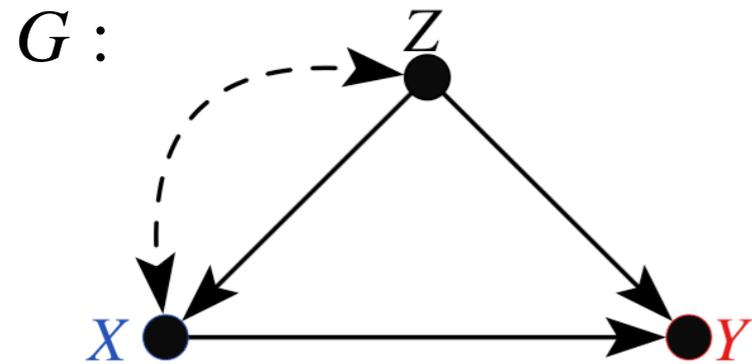


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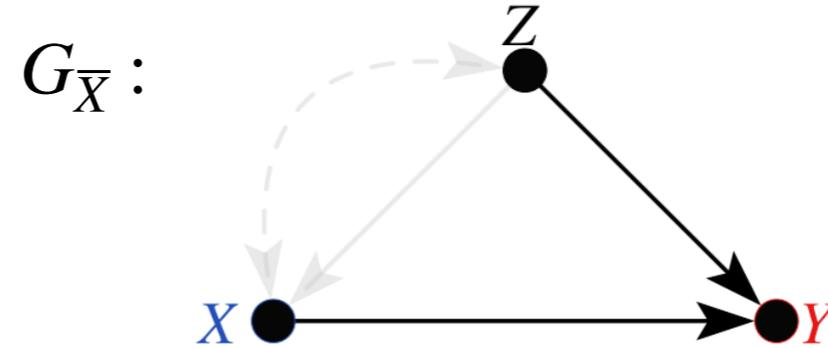
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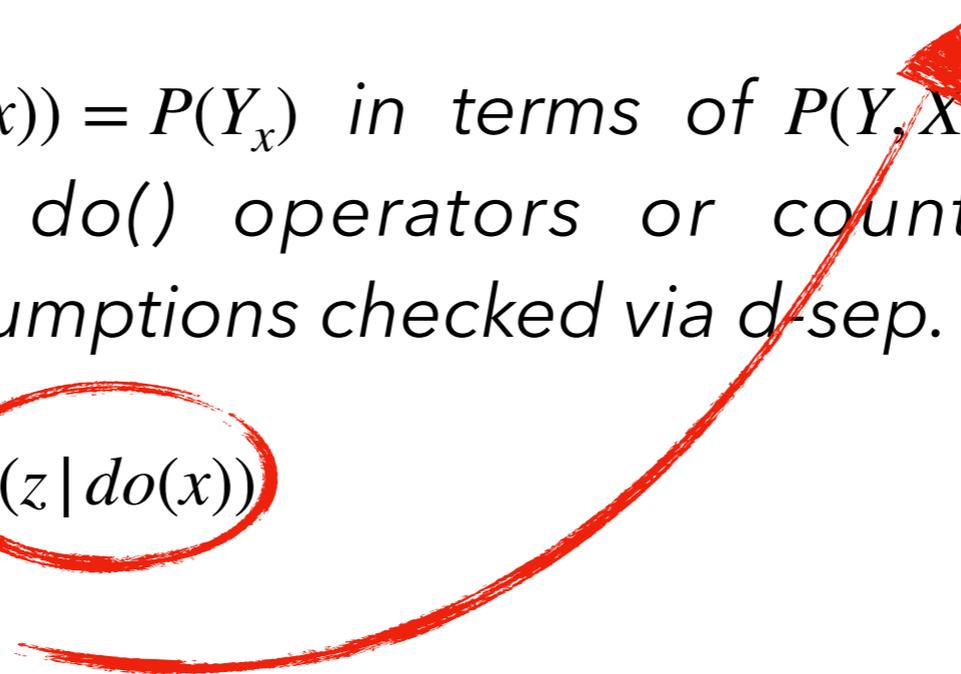


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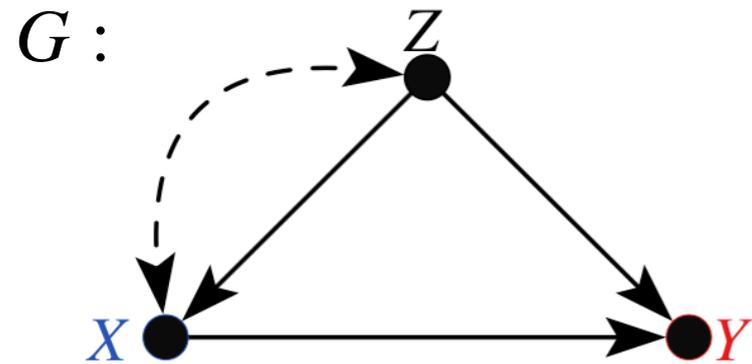
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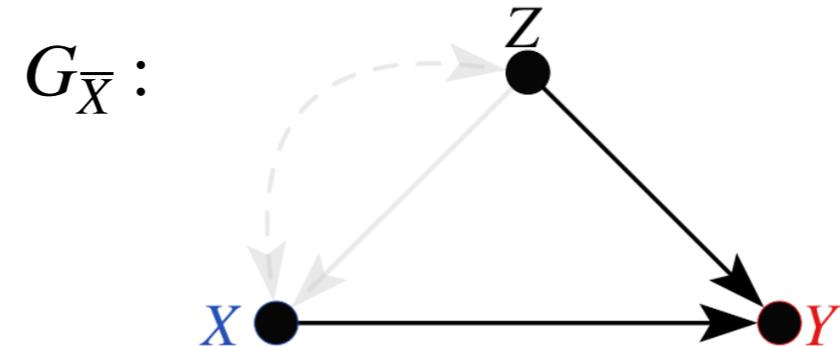


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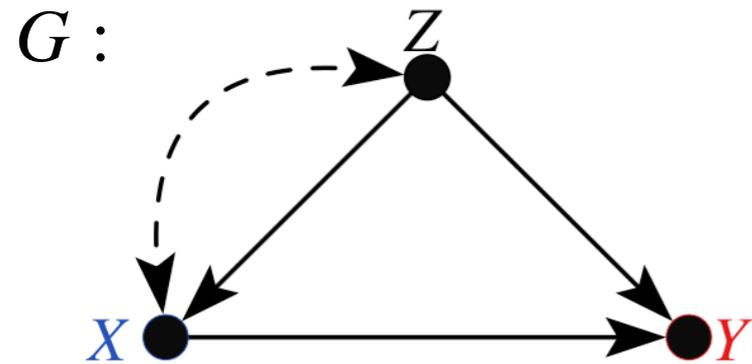


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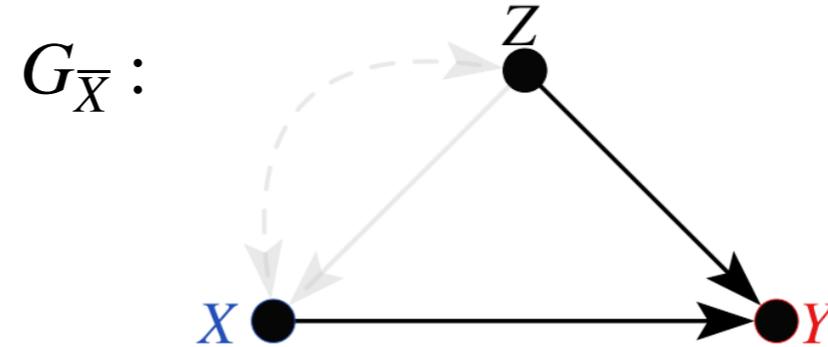
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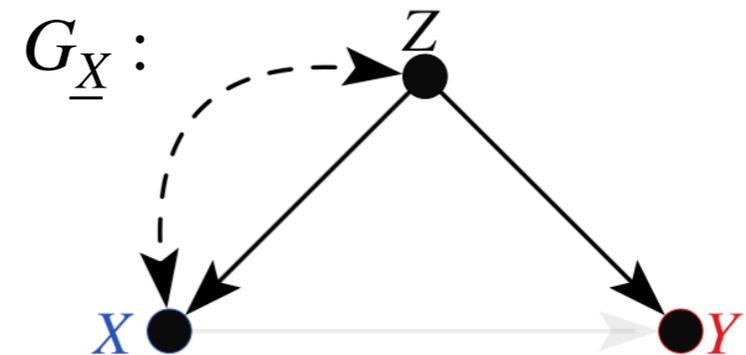


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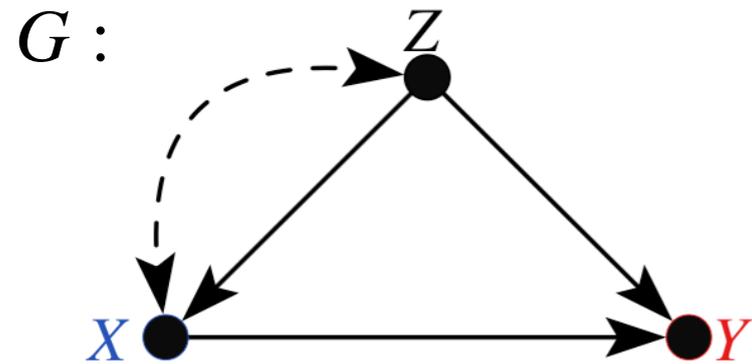
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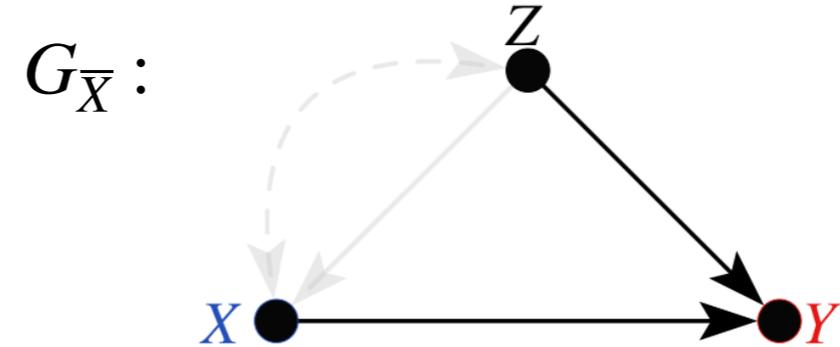


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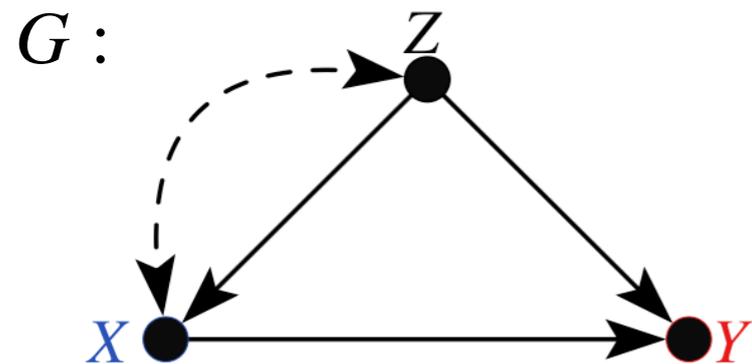


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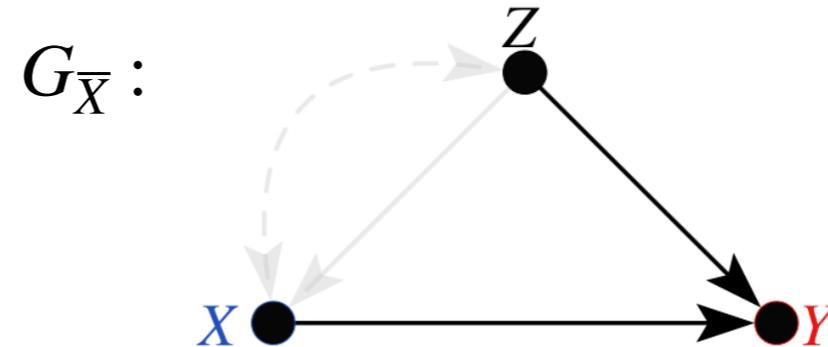
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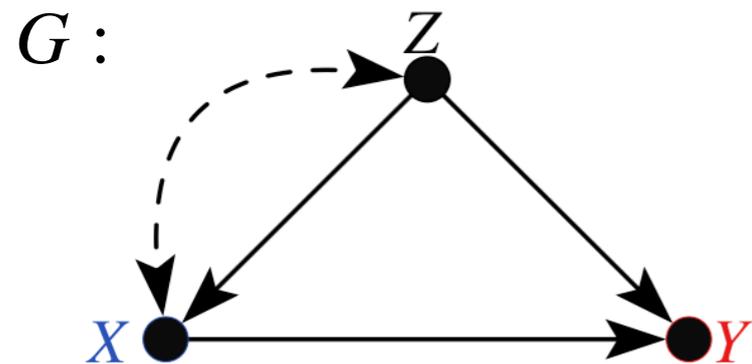
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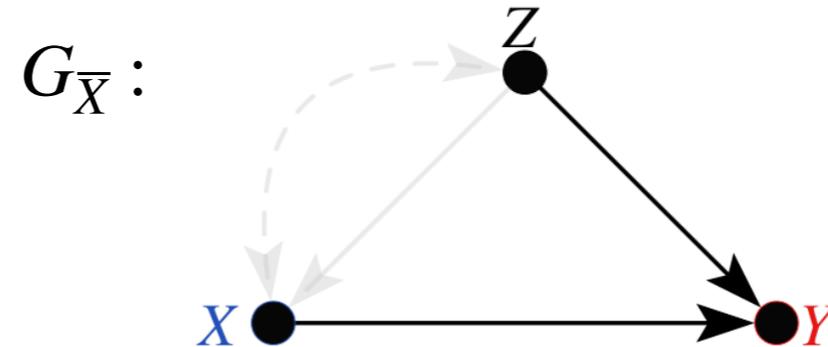
$$\begin{aligned}
 (Y \perp\!\!\!\perp X|Z)_{G_{\bar{X}}} &\implies Y_x \perp\!\!\!\perp X|Z_x \\
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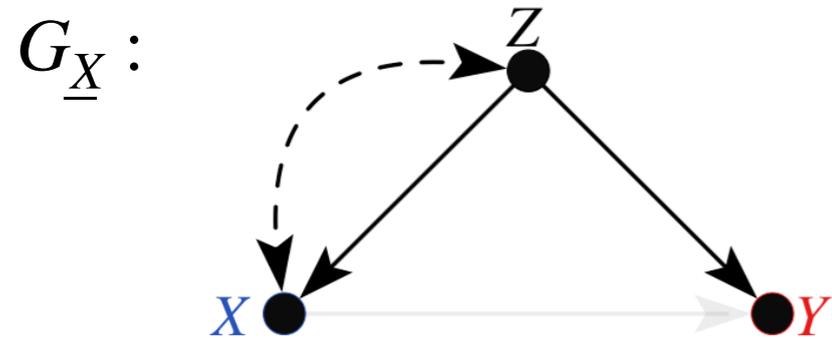
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Complete solution: do-calculus

The previous derivation showcases the (simplified) manipulation rules you need to know for massaging causal expressions (+ basic probability theory).

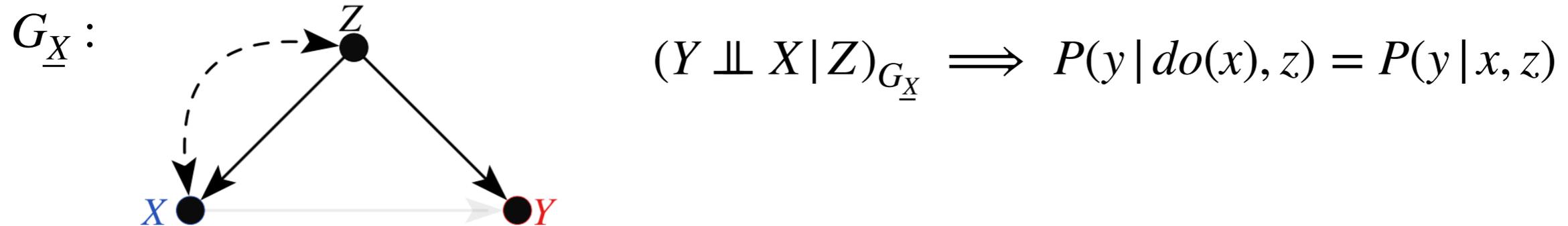
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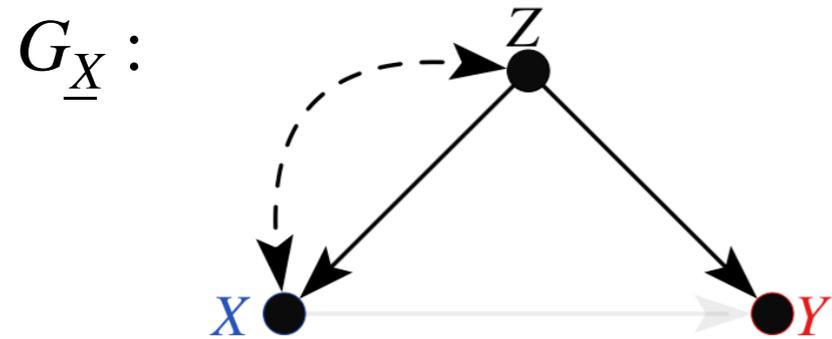
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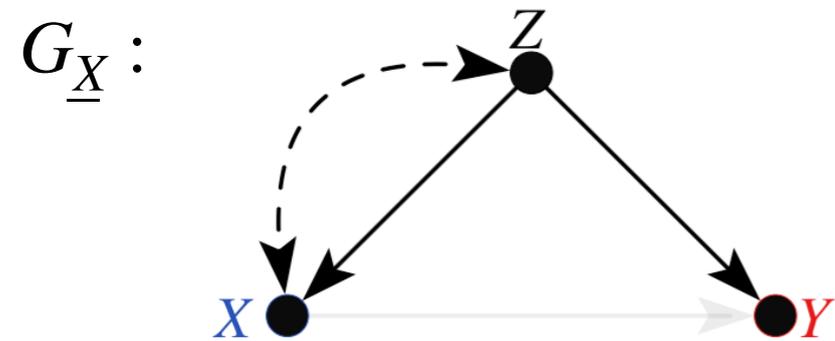


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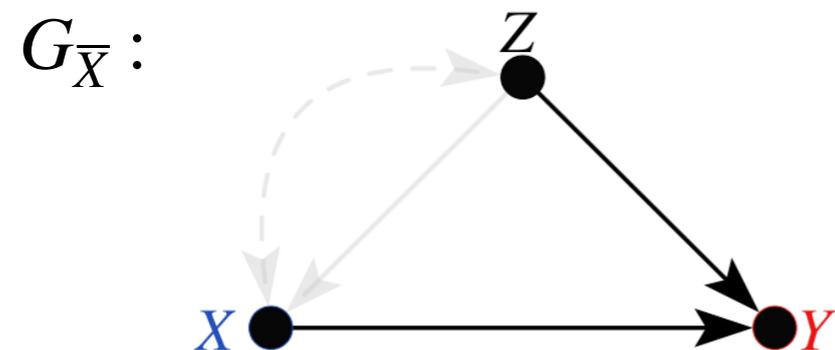
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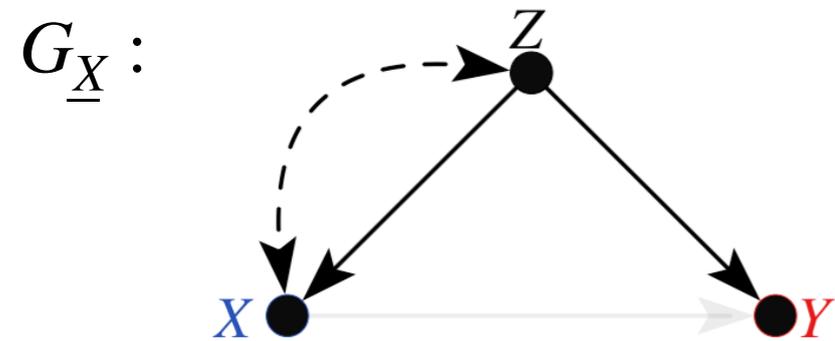
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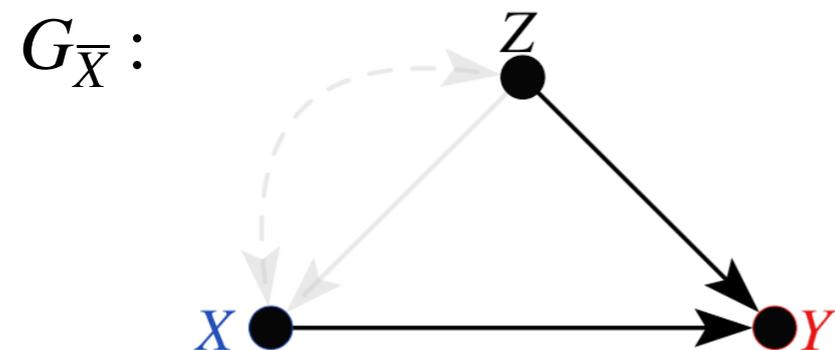
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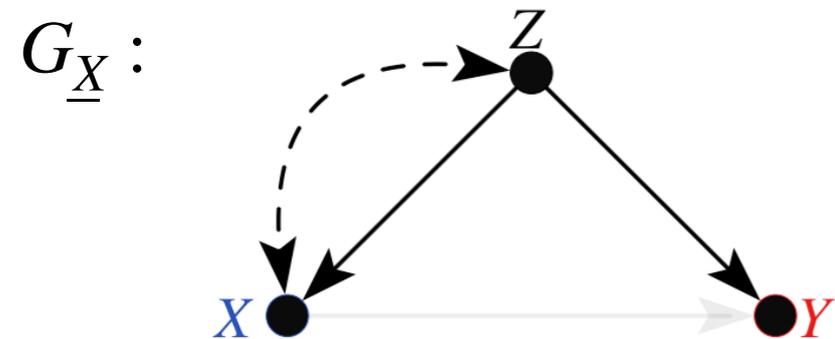
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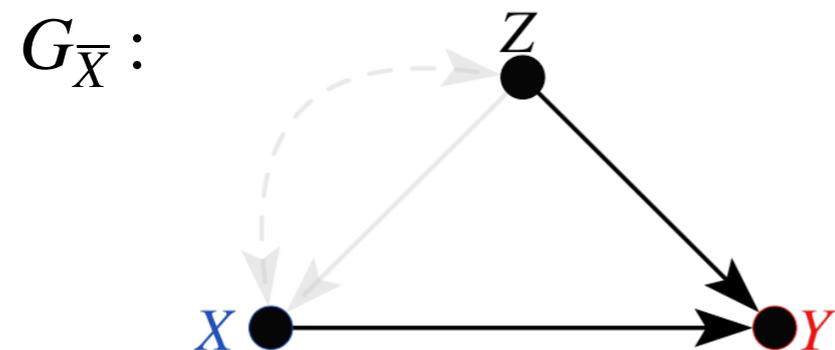
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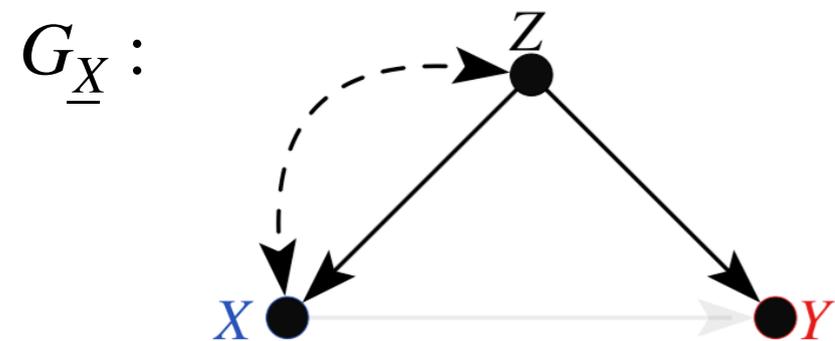


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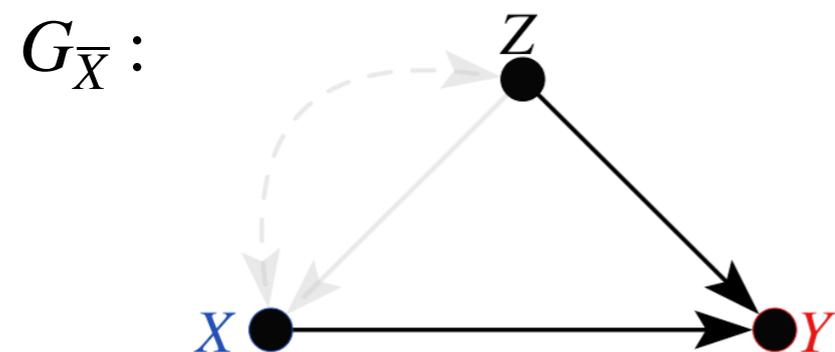
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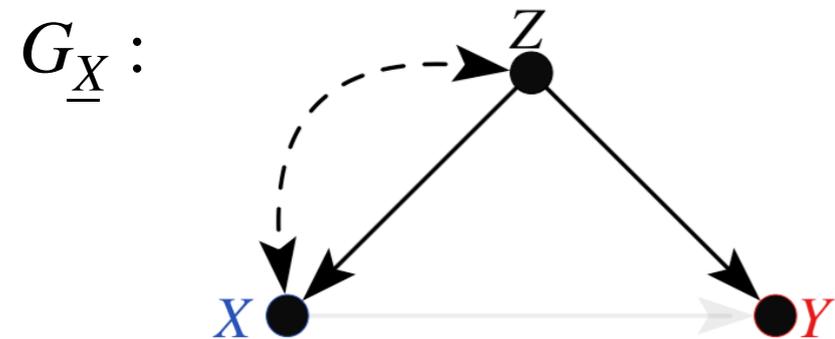
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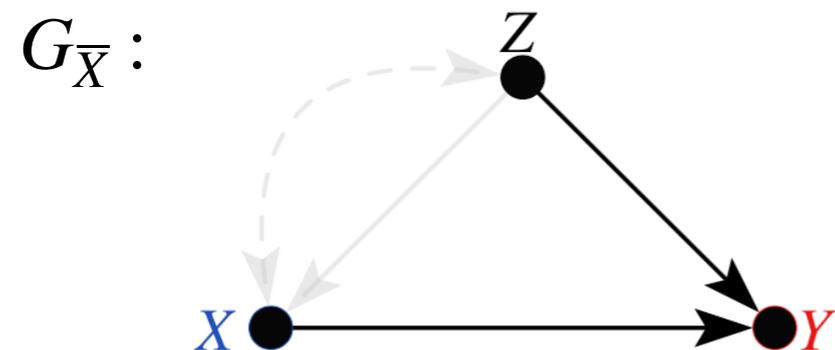
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We also have **complete algorithms**: completeness assures us that, if we can't find a solution, it is impossible to identify the effect **without extra assumptions**. That is, no other method can do better.

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Our goal: extend our modeling tools to formally characterize ***when*** and ***how***.

Transportability

(exp/obs) dist pop A, B, ... → (exp/obs) dist target pop

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How can we operationalize this?

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- Need to **encode disparities/commonalities** between environments
- Our approach will be **nonparametric**, requiring only a **qualitative description of which mechanisms are suspected to be different**

Encoding disparities: selection nodes

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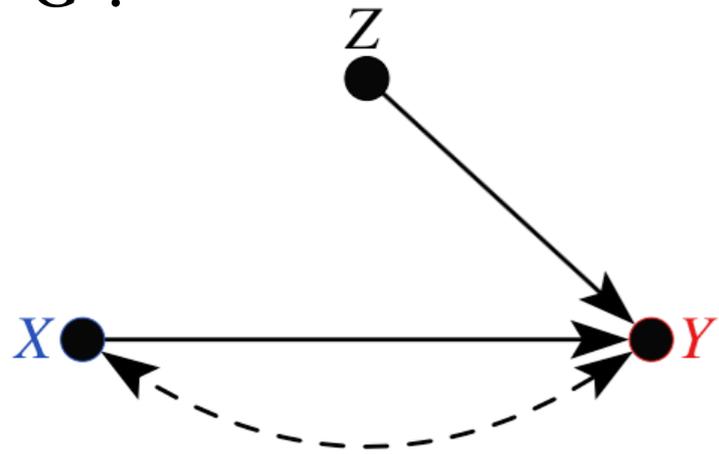
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Thus, **symbolically**, our task is to **remove conditioning on S on any $\text{do}()$ expression** (or counterfactual expression), since we do not have experimental data on the target domain.

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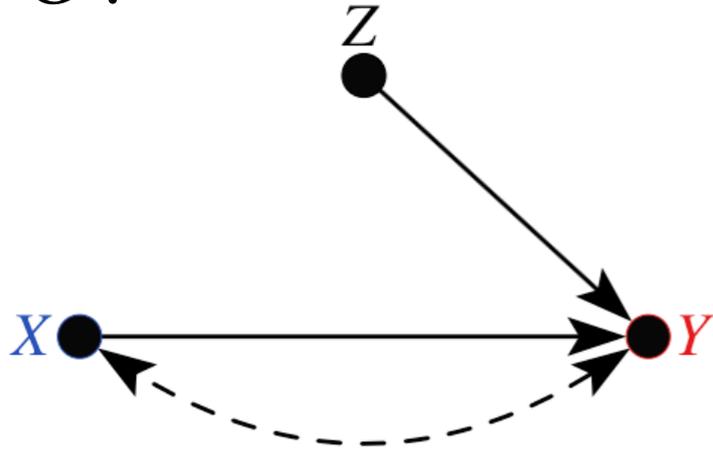
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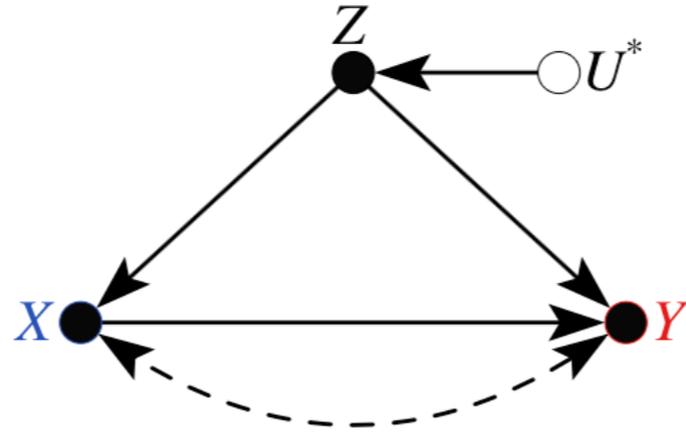


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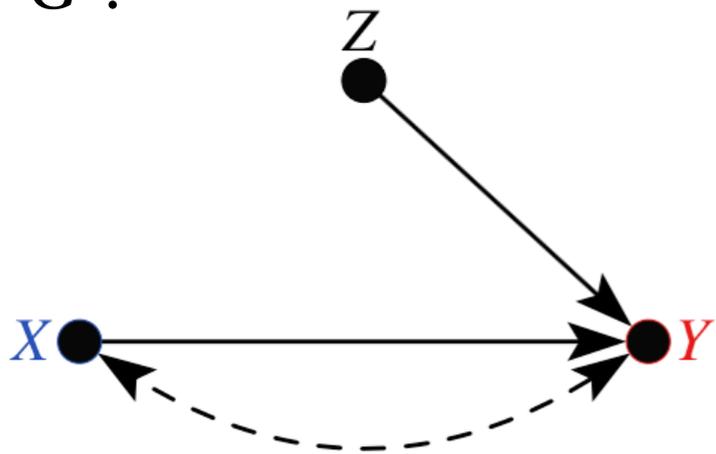


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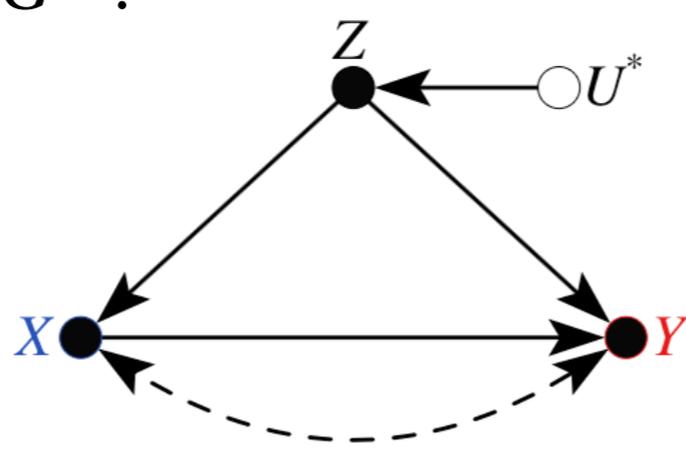


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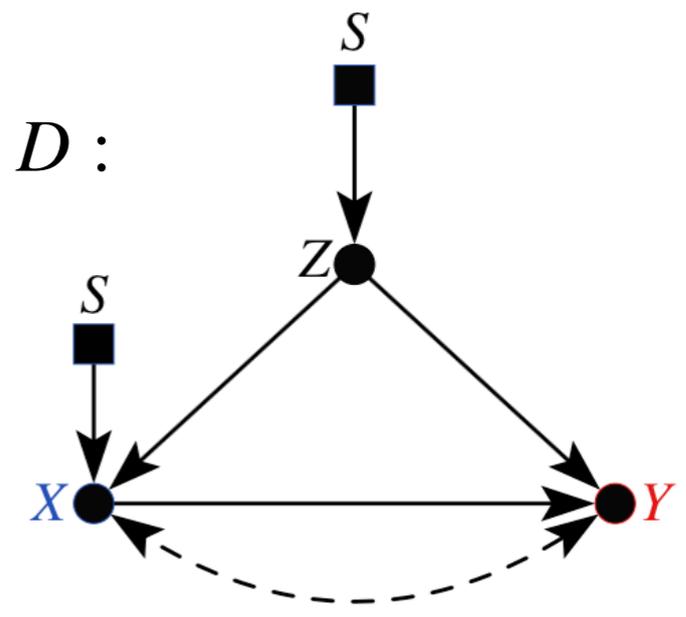
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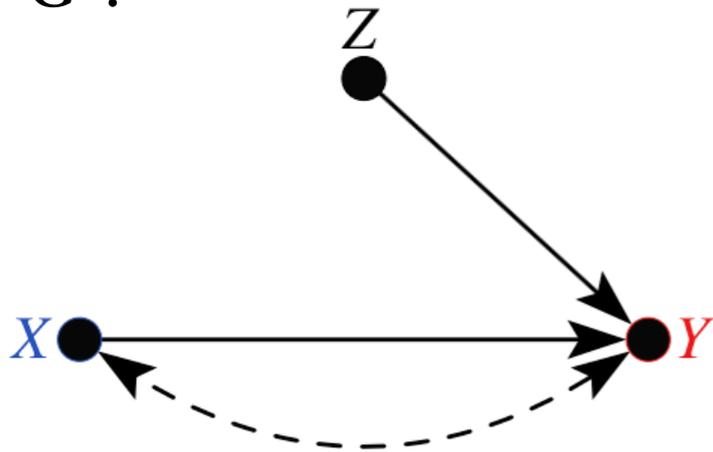


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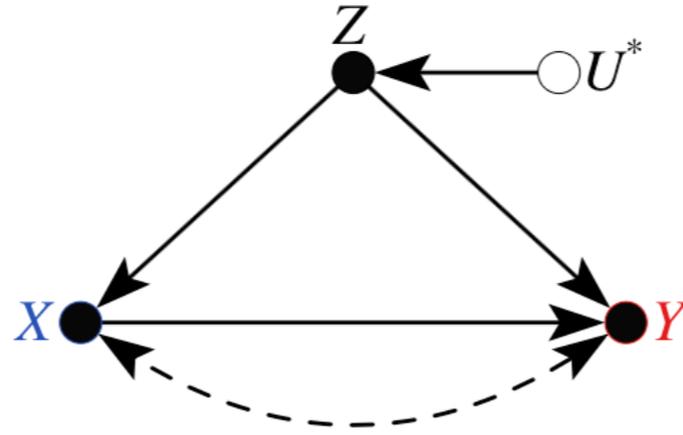


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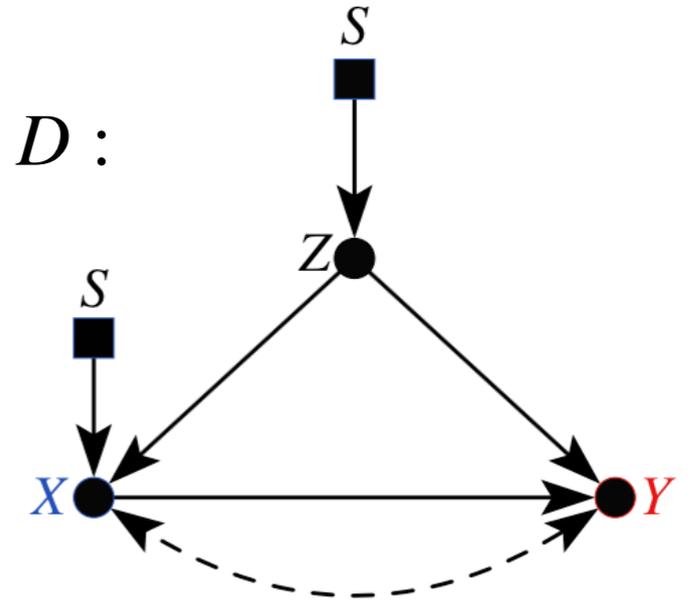
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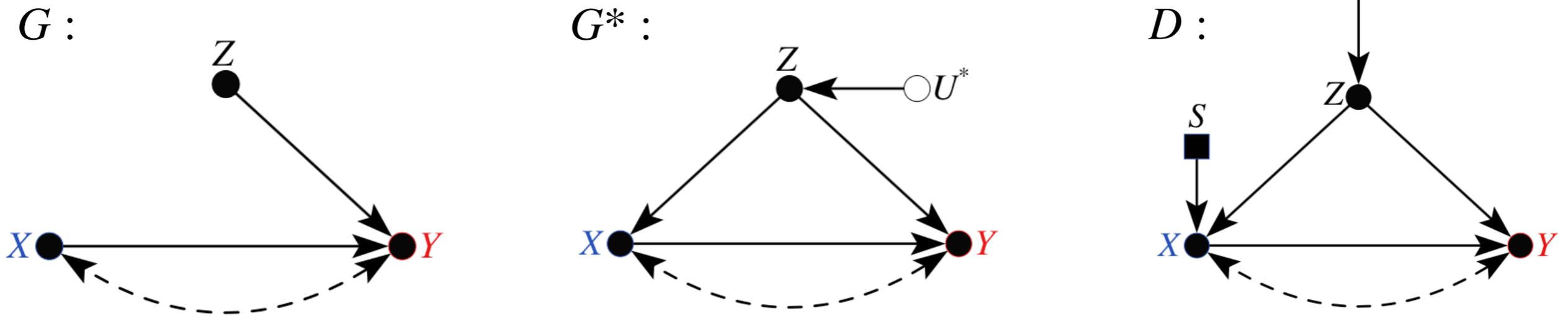


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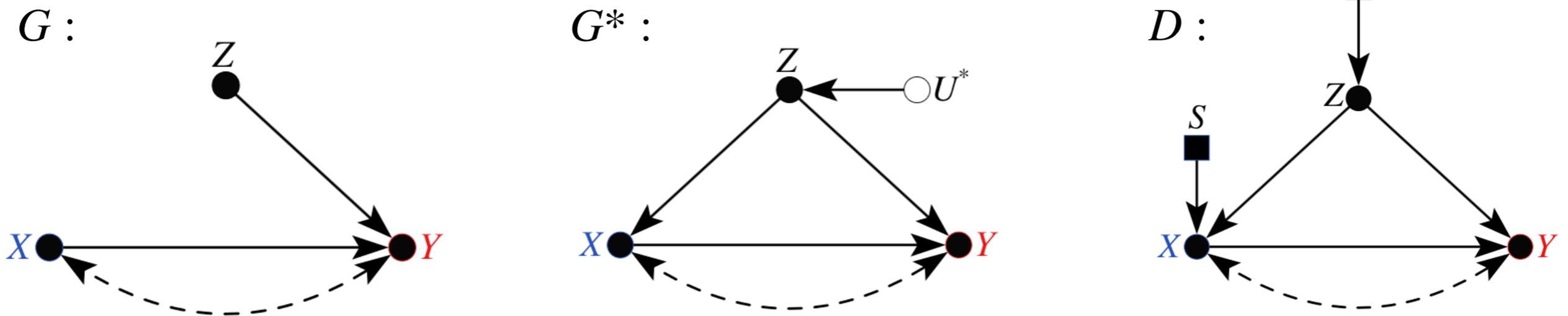
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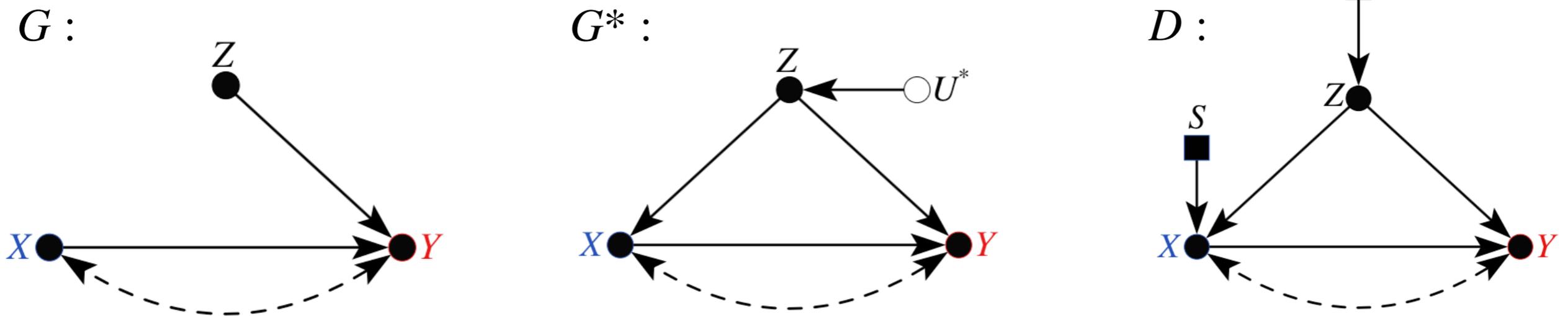


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For clarity, selection nodes (S) are represented by square nodes (■).

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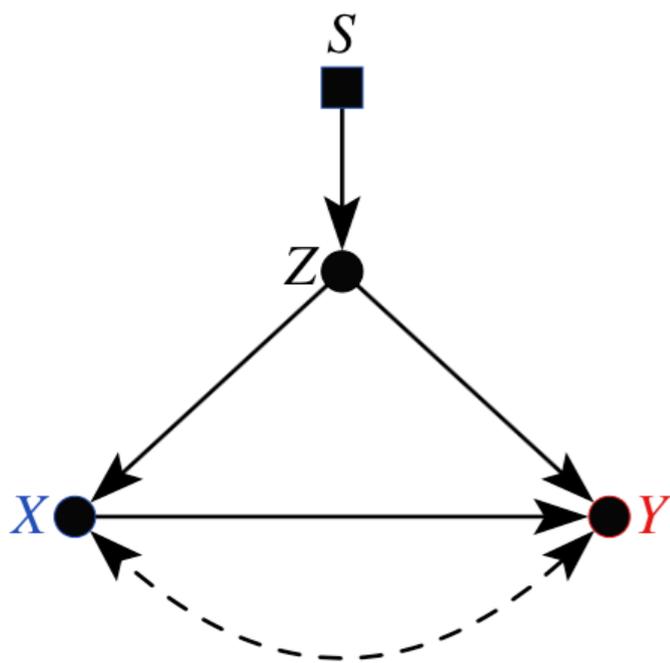
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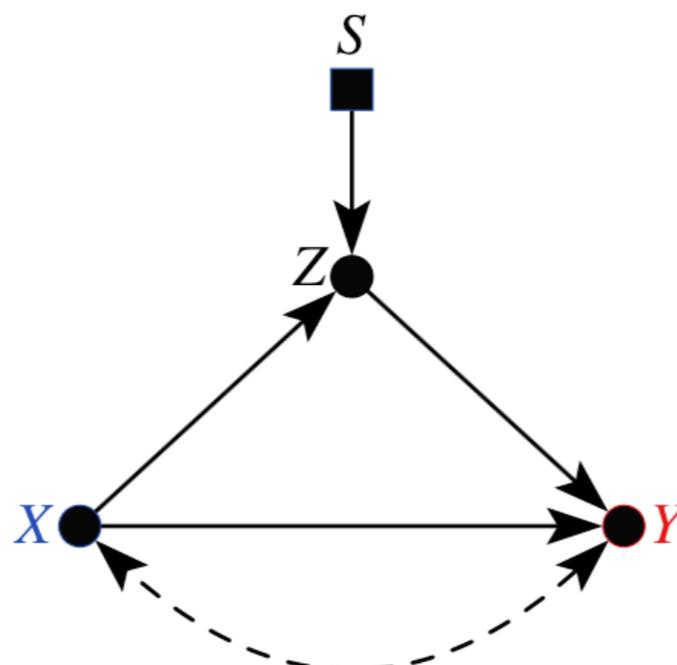
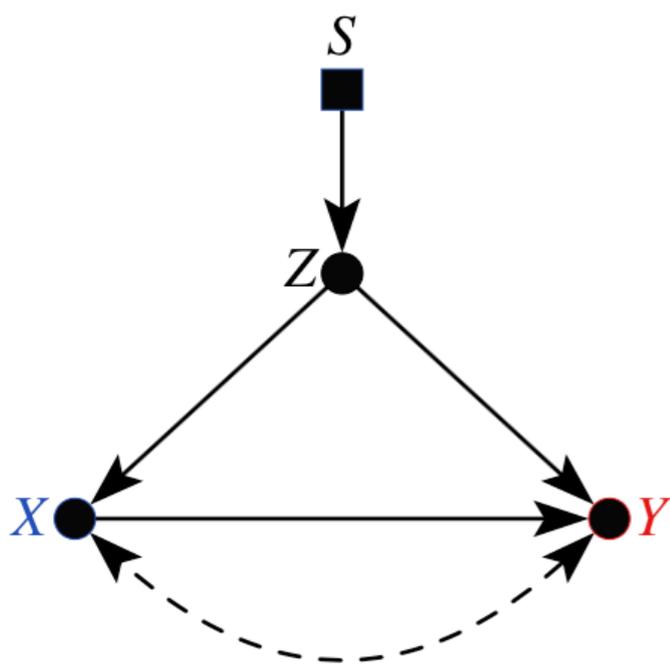
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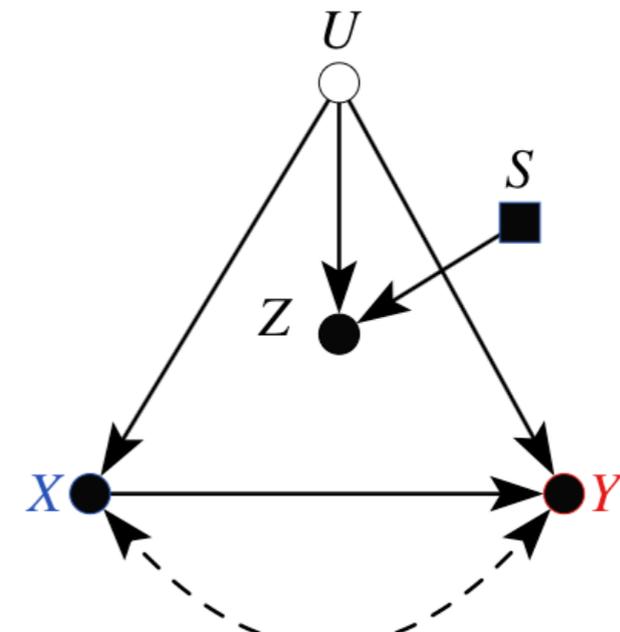
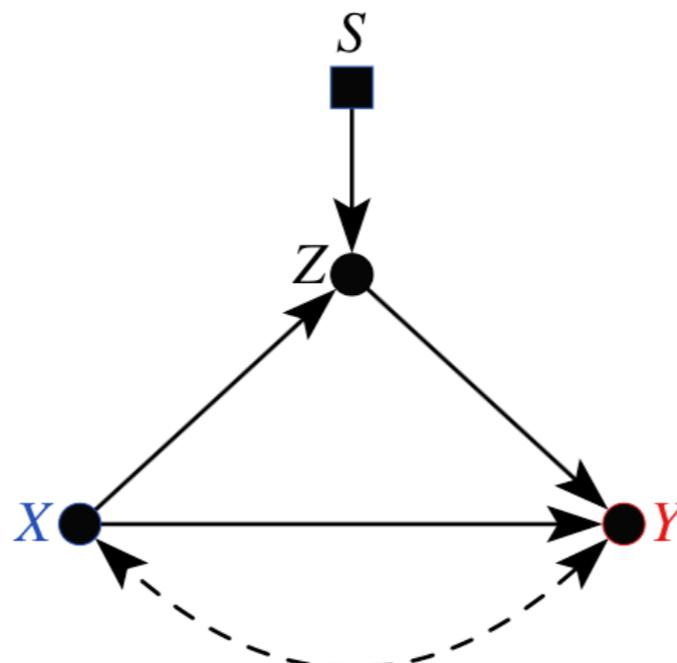
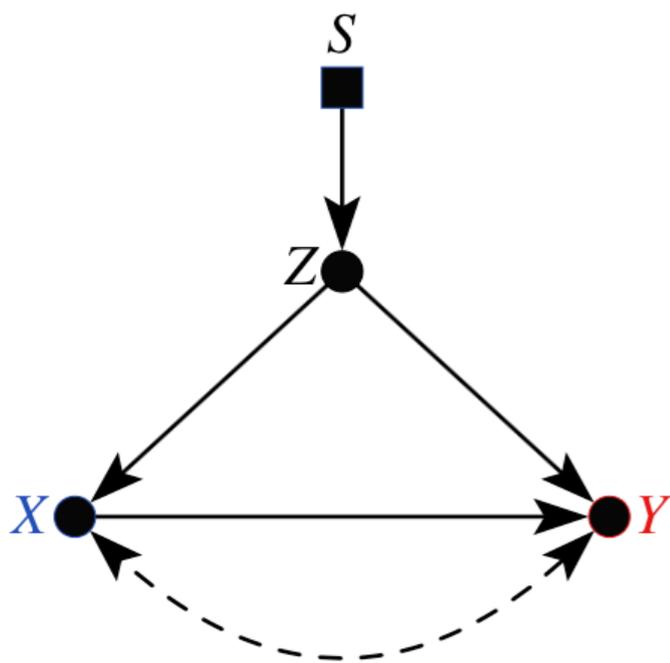
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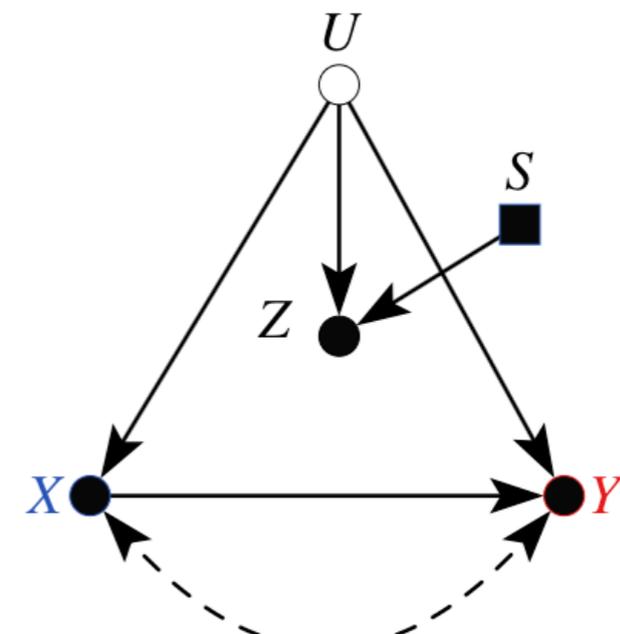
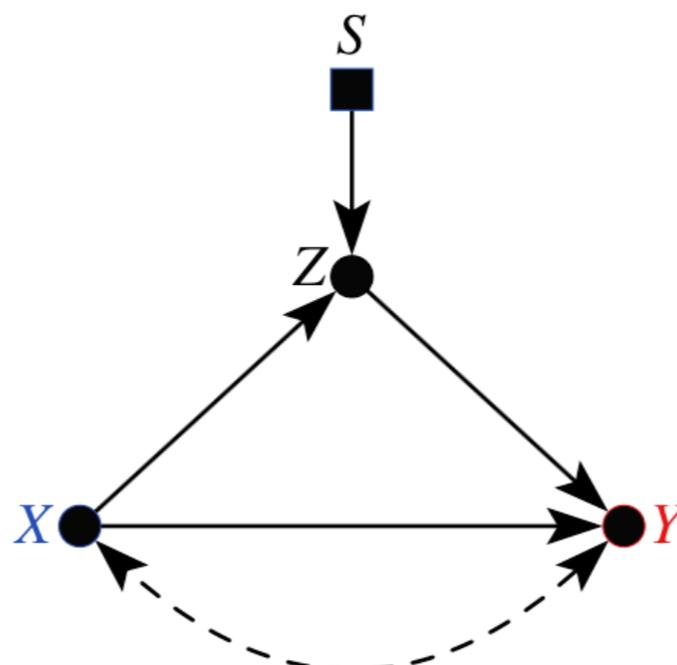
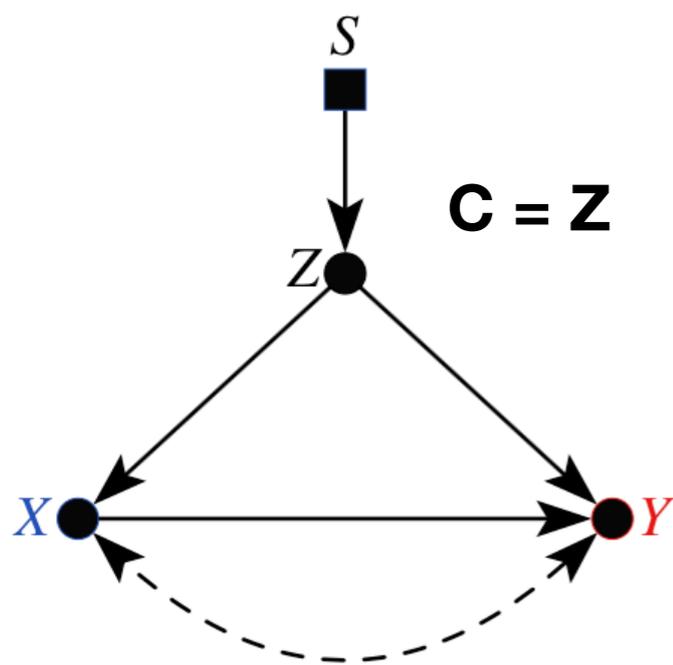
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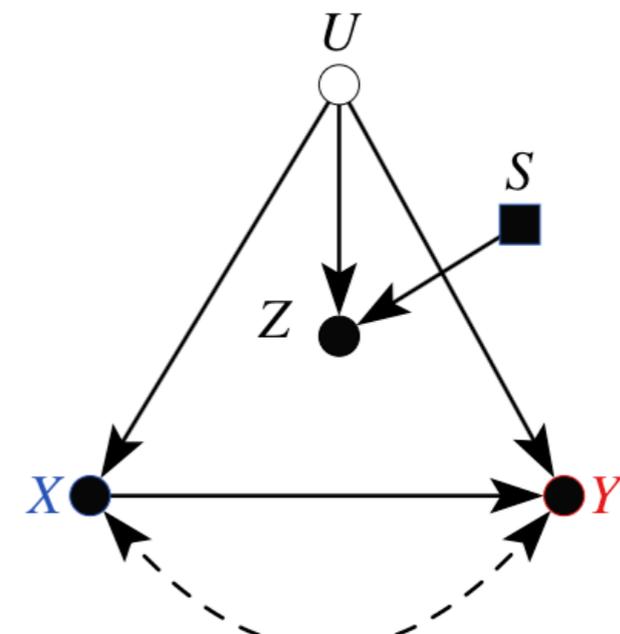
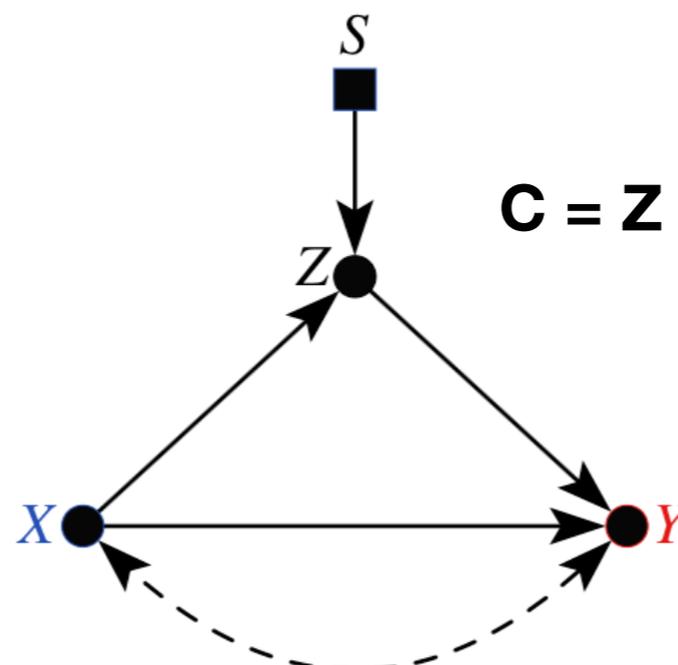
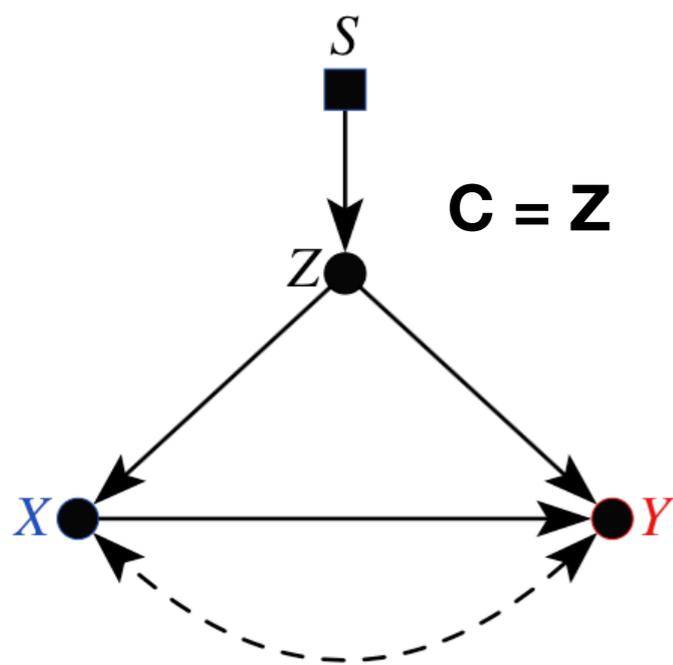
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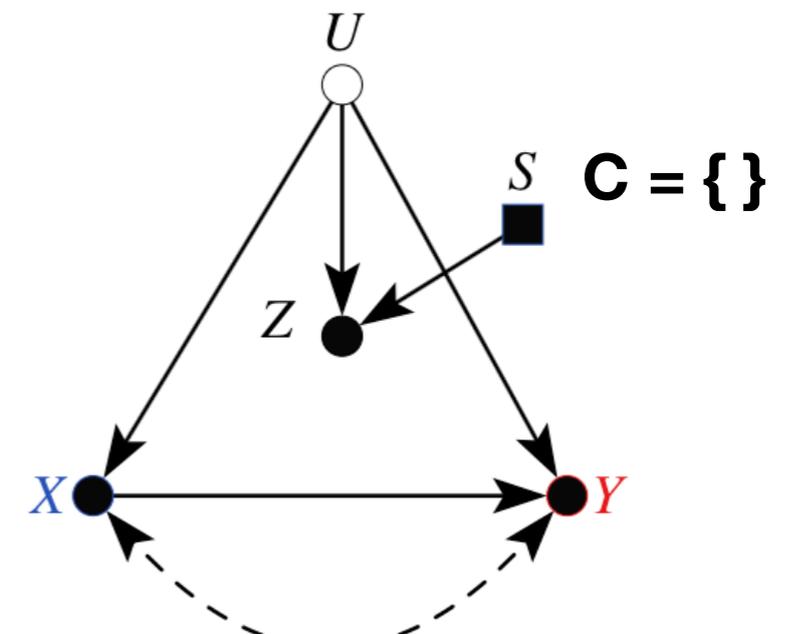
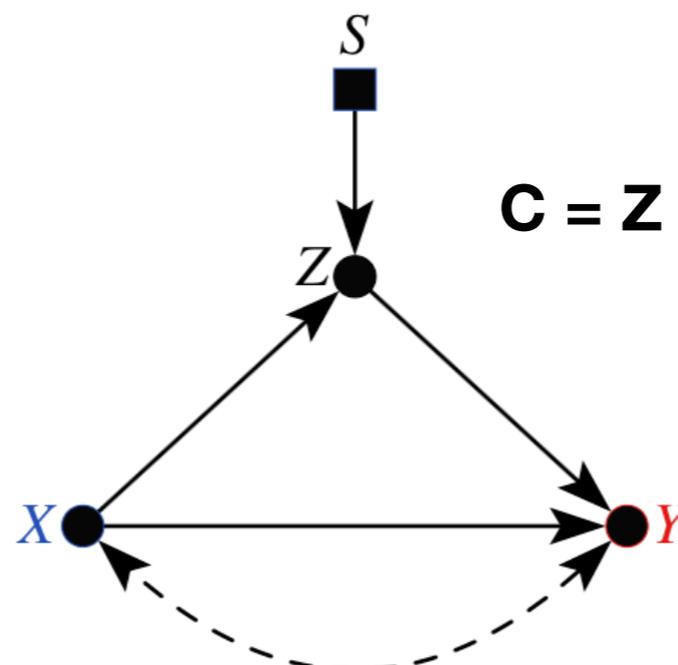
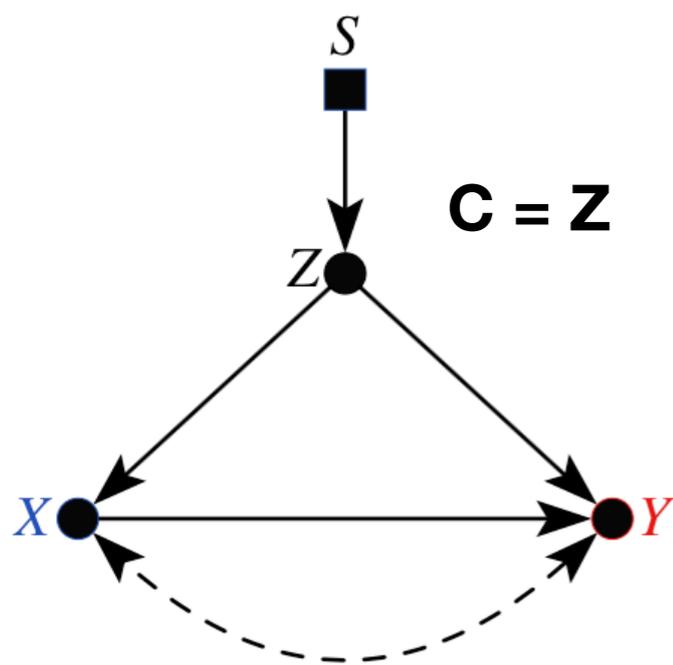
2) Direct transportability

- Transportable directly from source to target (Manski called "external validity")

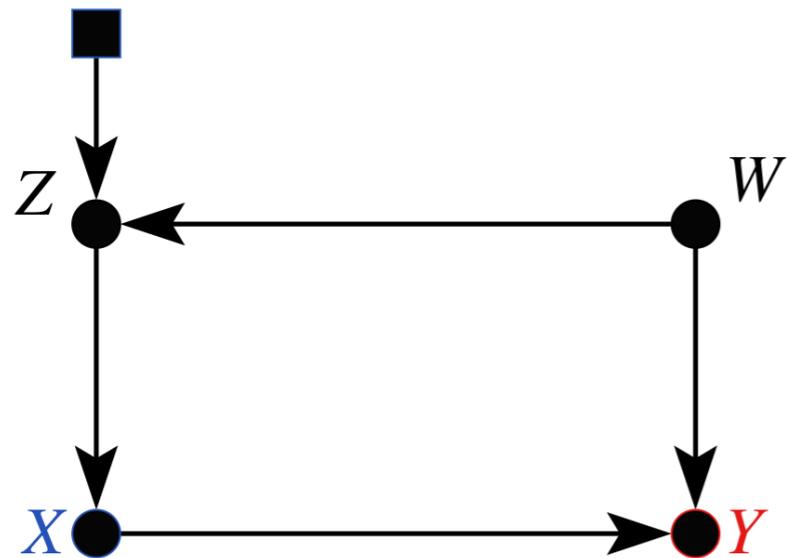
$$E^*[Y | do(x), z] = E[Y | do(x), z]$$

- Reduced to checking d-separation in selection diagram

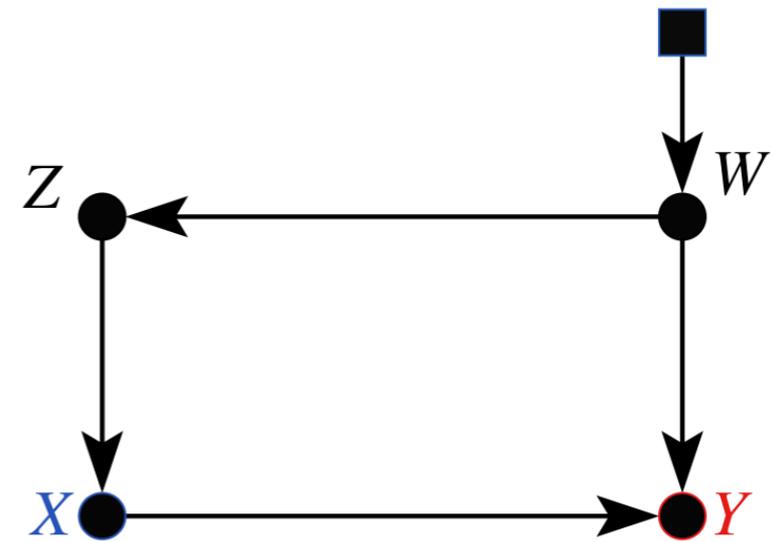
$$(Y \perp\!\!\!\perp S | C, X)_{D_{\bar{X}}} \implies E^*[Y | do(x), c] = E[Y | do(x), c, s] = E[Y | do(x), c]$$



Two lessons

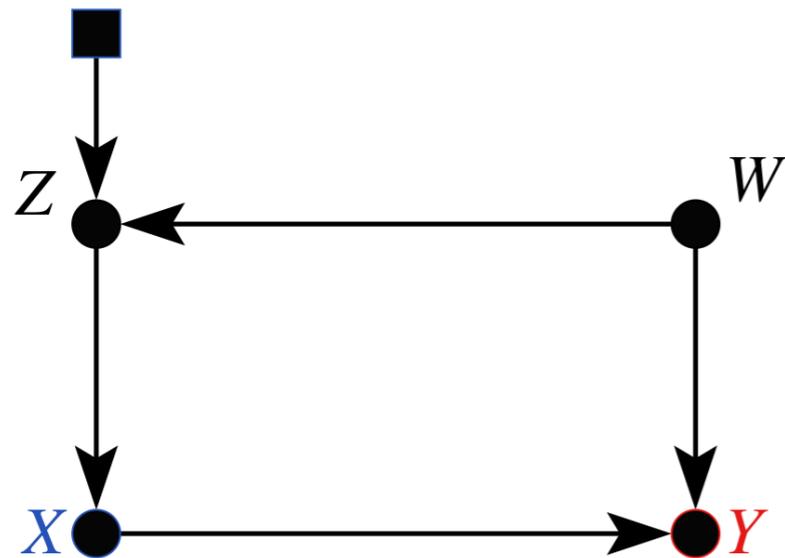


VS

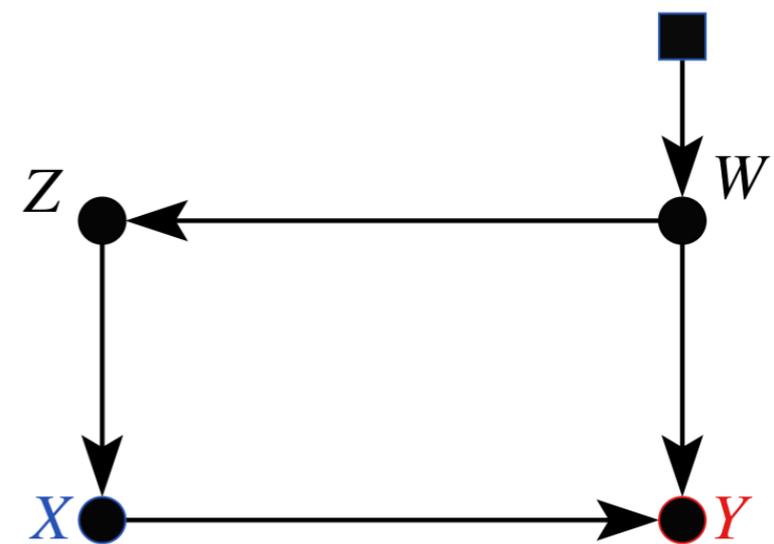


Both Z and W are valid adjustments for the identification of $P(y|\text{do}(x))$. But are they equally important for transporting the effect to Π^* ? (hint: use d-sep.)

Two lessons



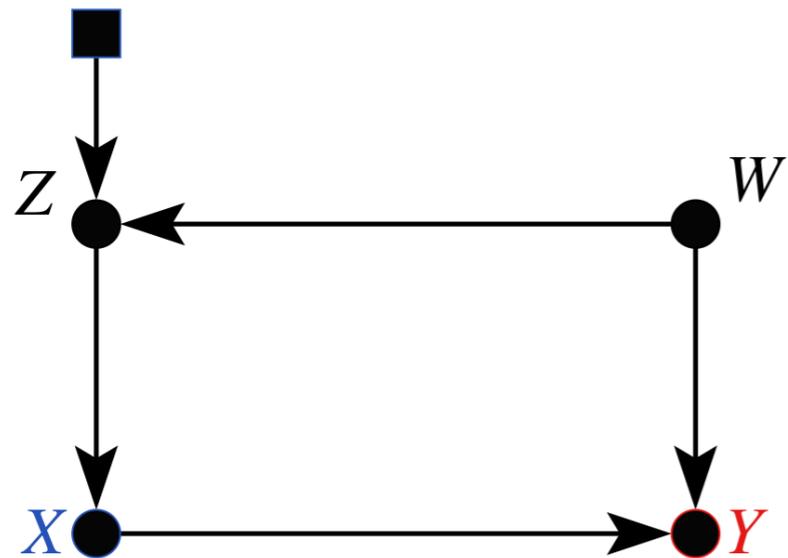
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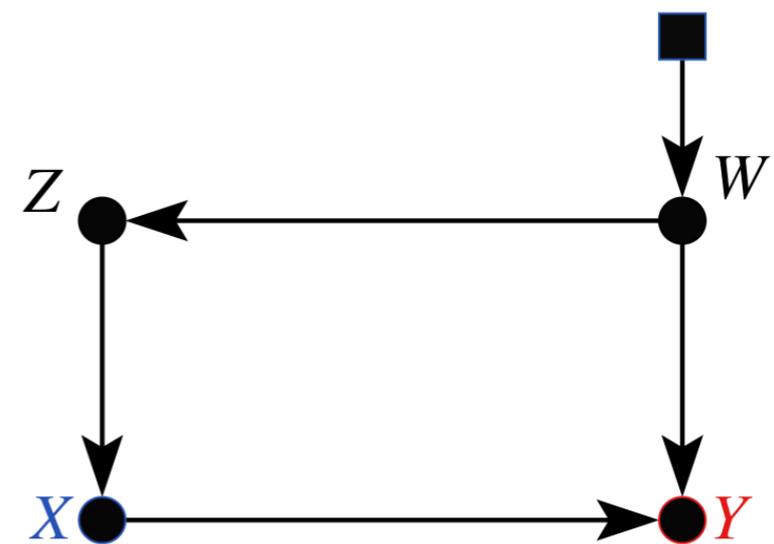
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Two lessons



VS



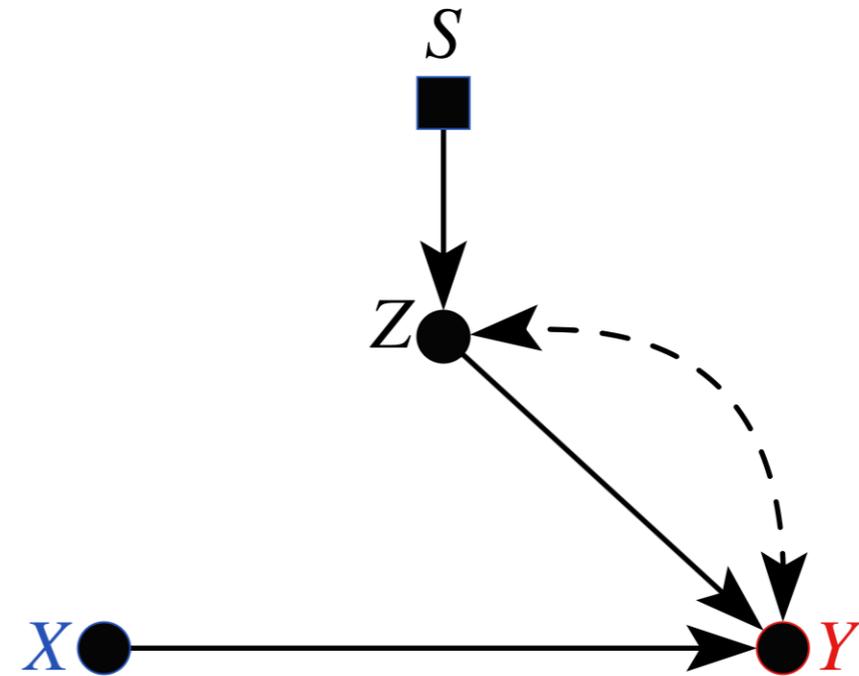
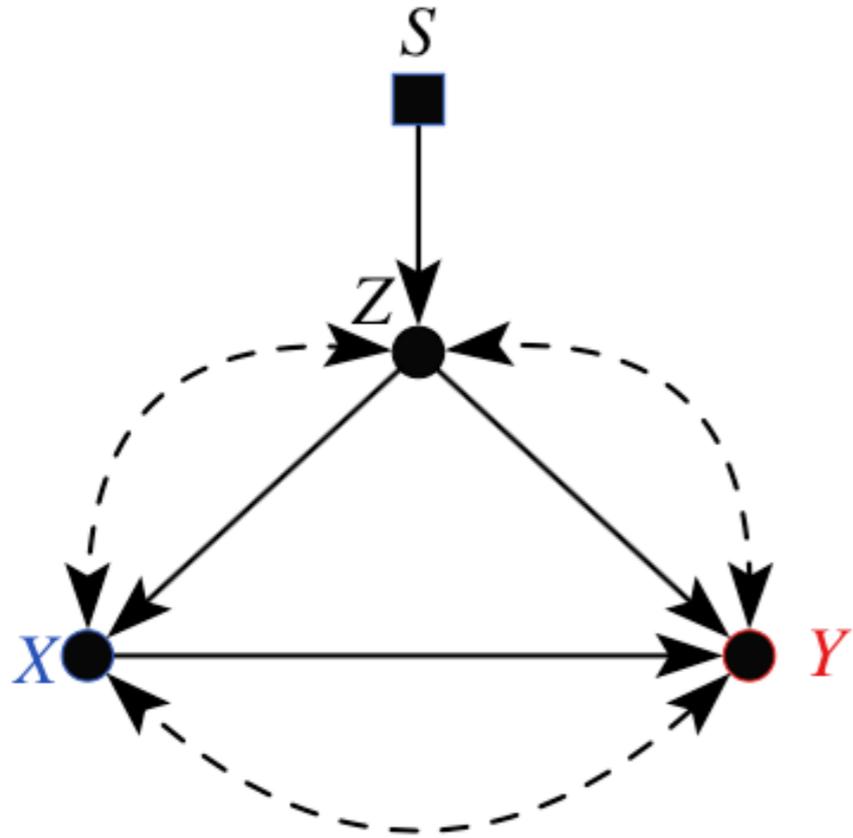
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Lesson 1: differences in propensity to receive treatment do not matter for transportability of causal effects. What matters are potential effect-modifiers.

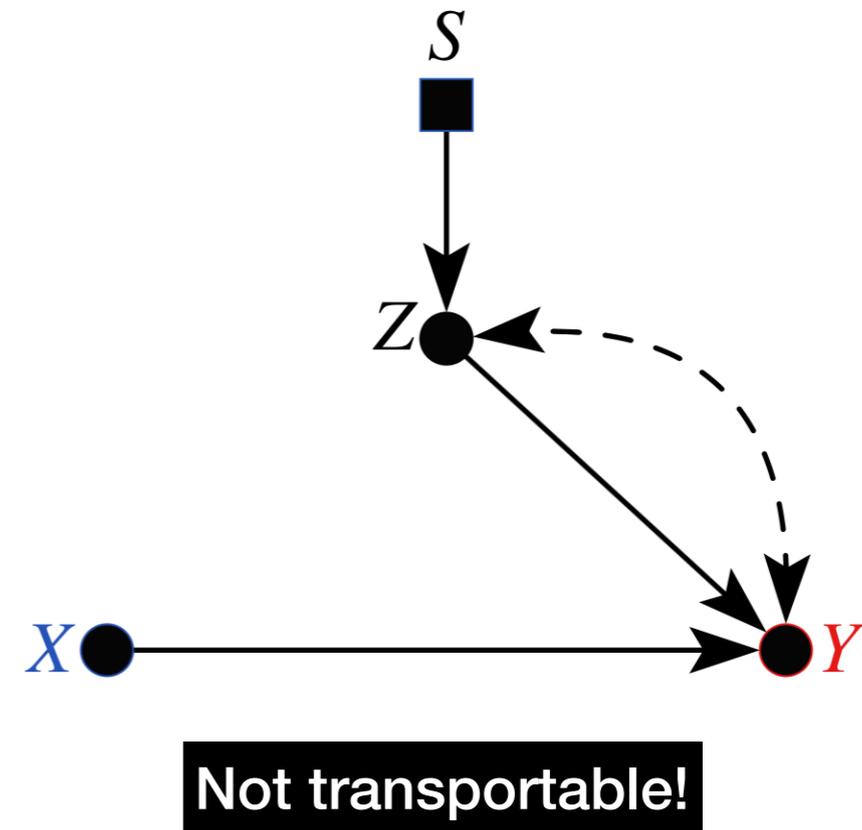
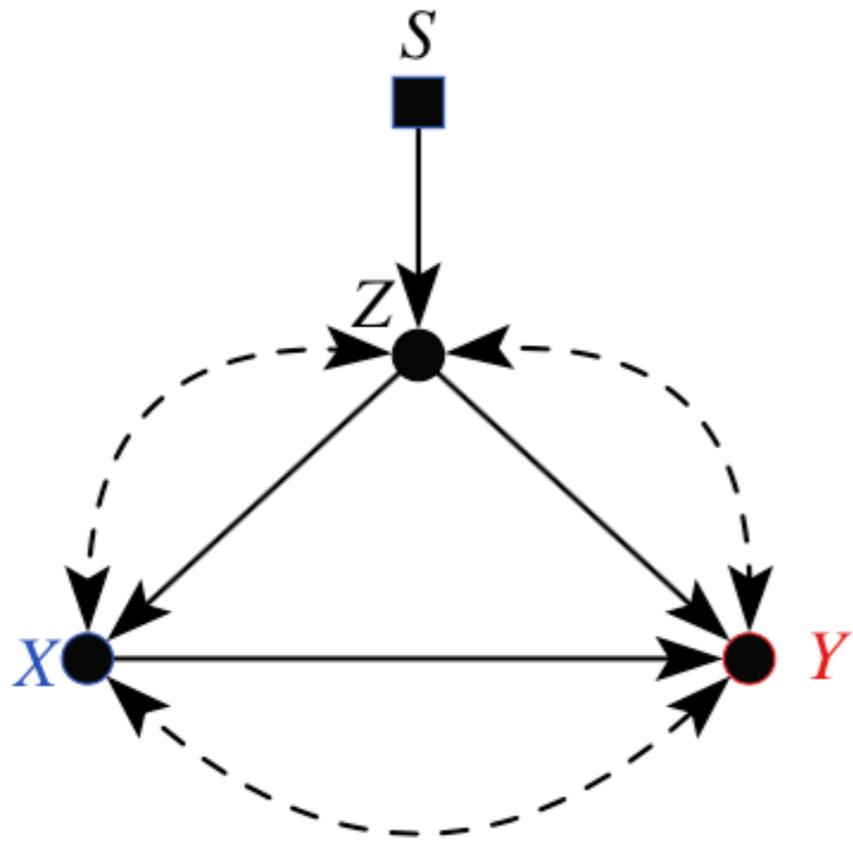
Two lessons

Is a randomized control trial really a gold standard?



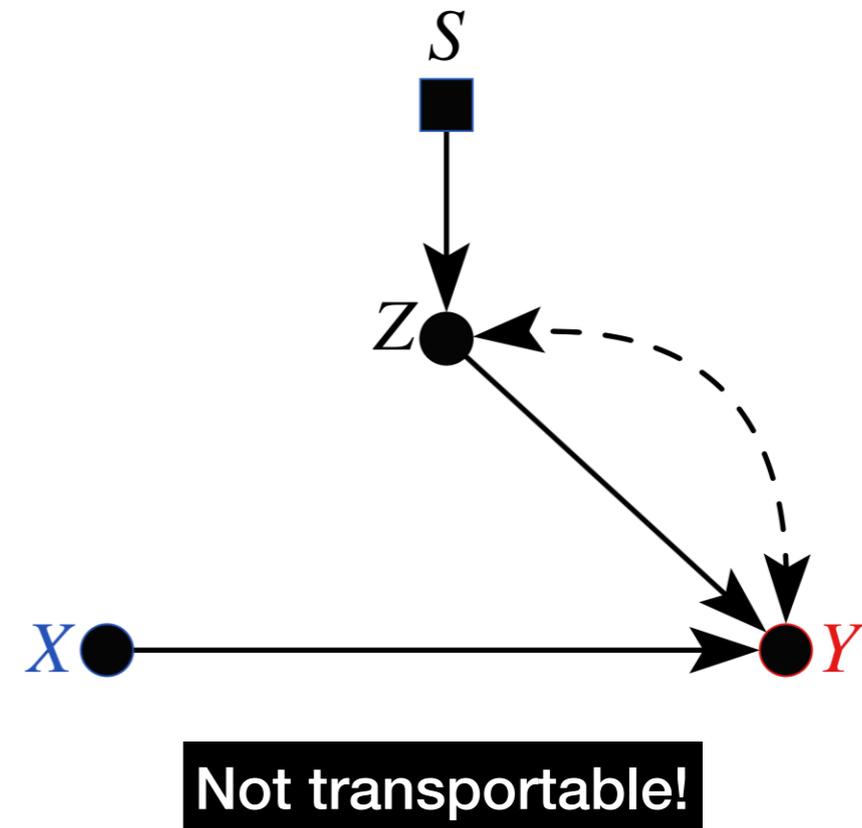
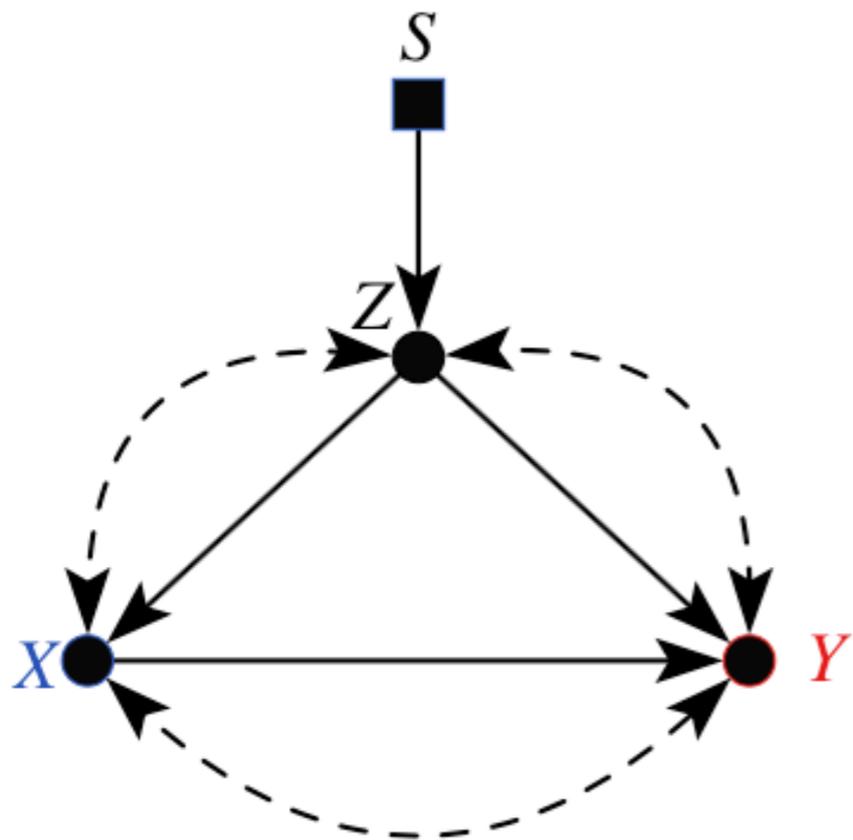
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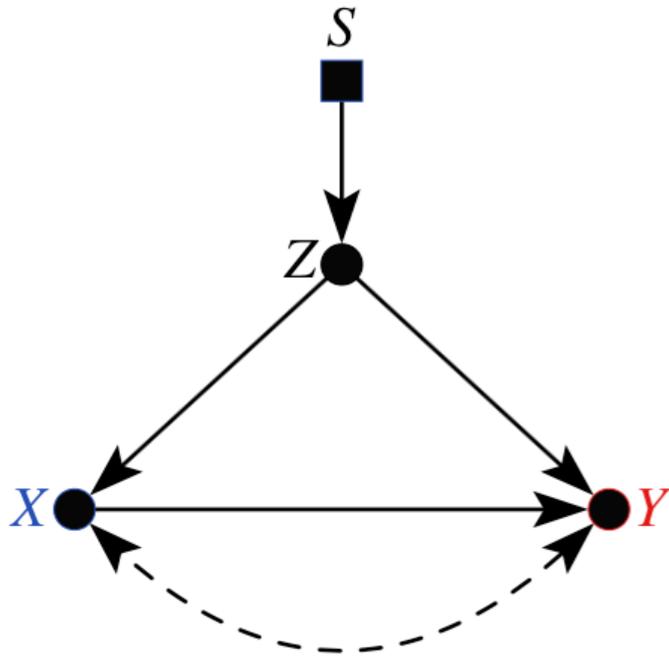
Lesson 2: unless one wants to confine experimental results to the strict conditions of the studied subpopulation, even with a perfect RCT one still needs to go through a transportability exercise (ie, causal modeling).

Finding invariances: beyond direct transportability

- *Many effects are not directly TR, but are TR after proper adjustment.*
- **Strategy:** *break relations that are not directly TR to find invariant pieces.*

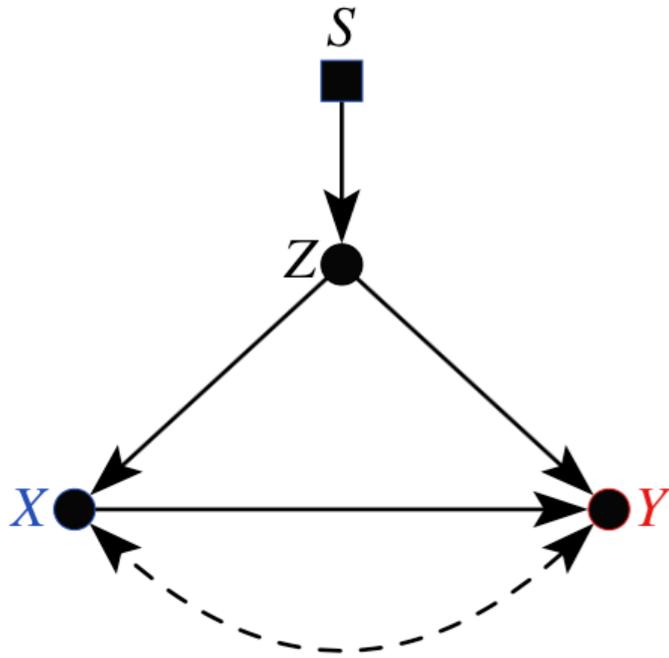
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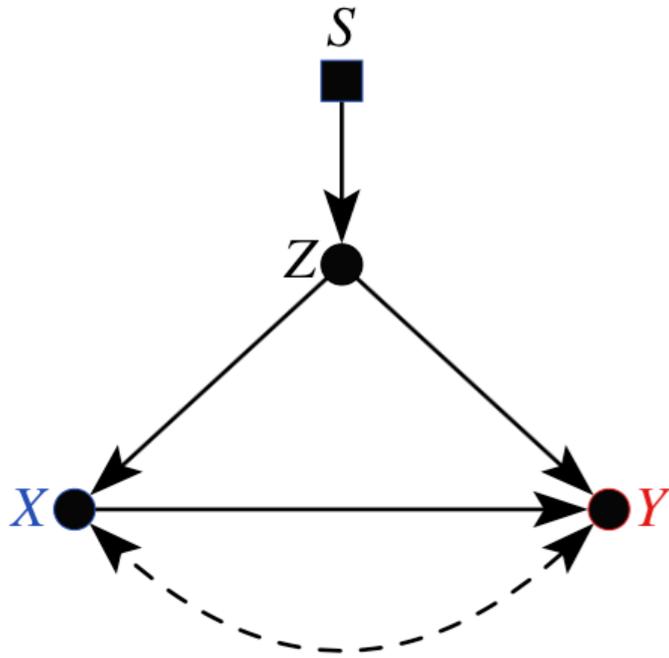
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$$\begin{aligned} E^*[Y | do(x)] &= E[Y | do(x), s] \\ &= \sum_z E[Y | do(x), z, s] P(z | do(x), s) \\ &= \sum_z E[Y | do(x), z, s] P(z | s) \\ &= \sum_z \underbrace{E[Y | do(x), z]}_{z\text{-specific effect from source}} \underbrace{P^*(z)}_{\text{weight from target dist.}} \end{aligned}$$

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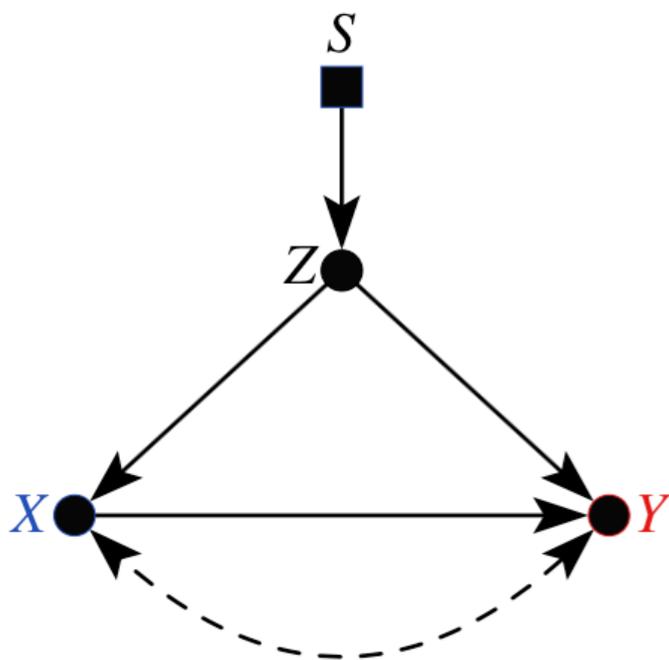
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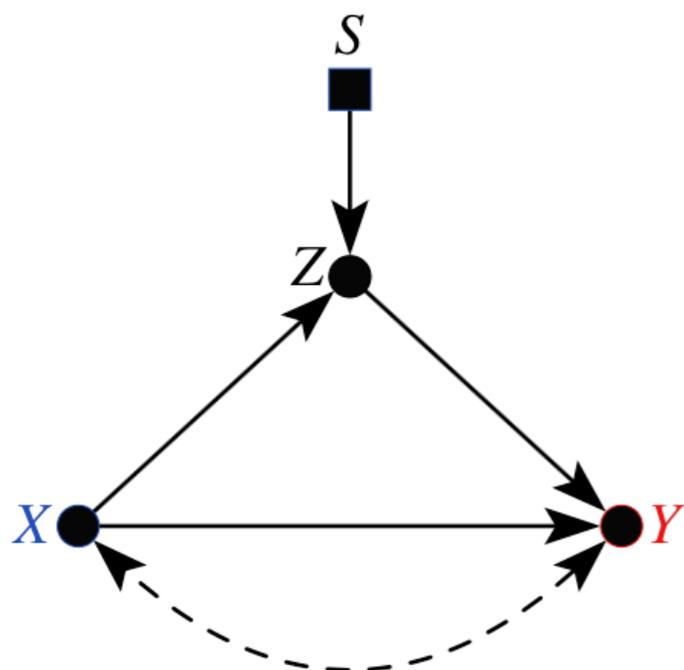


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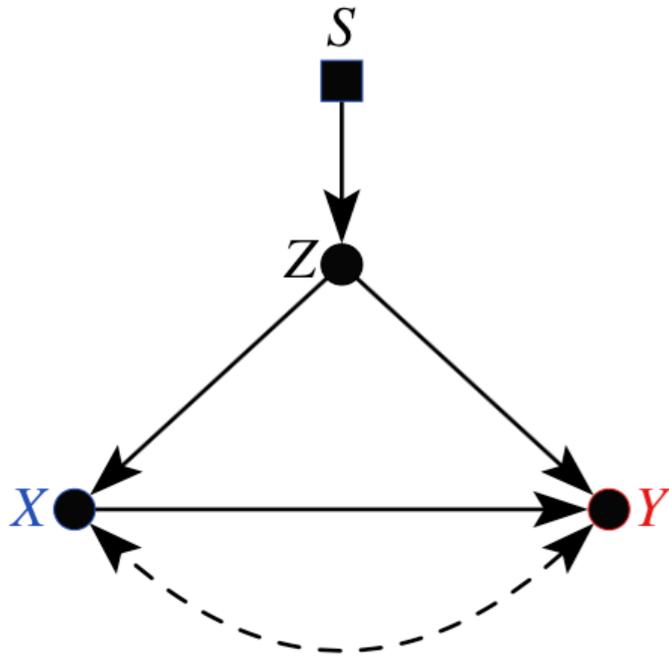
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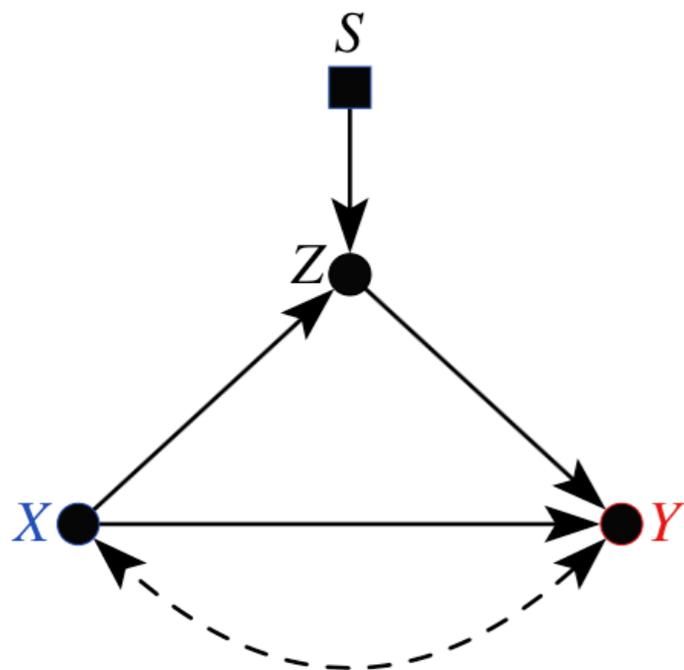


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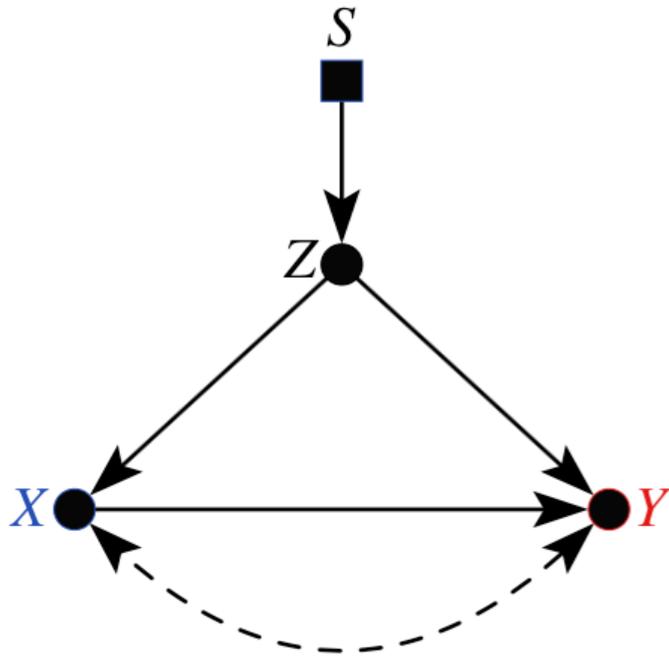
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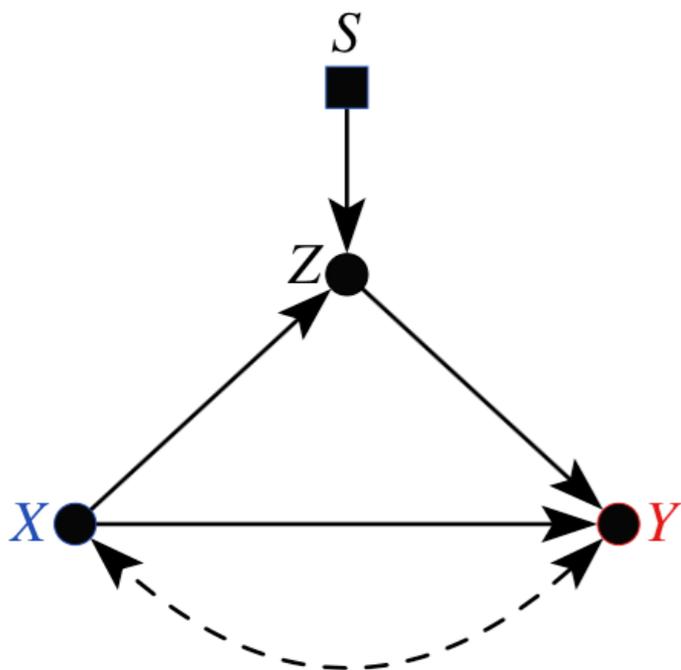
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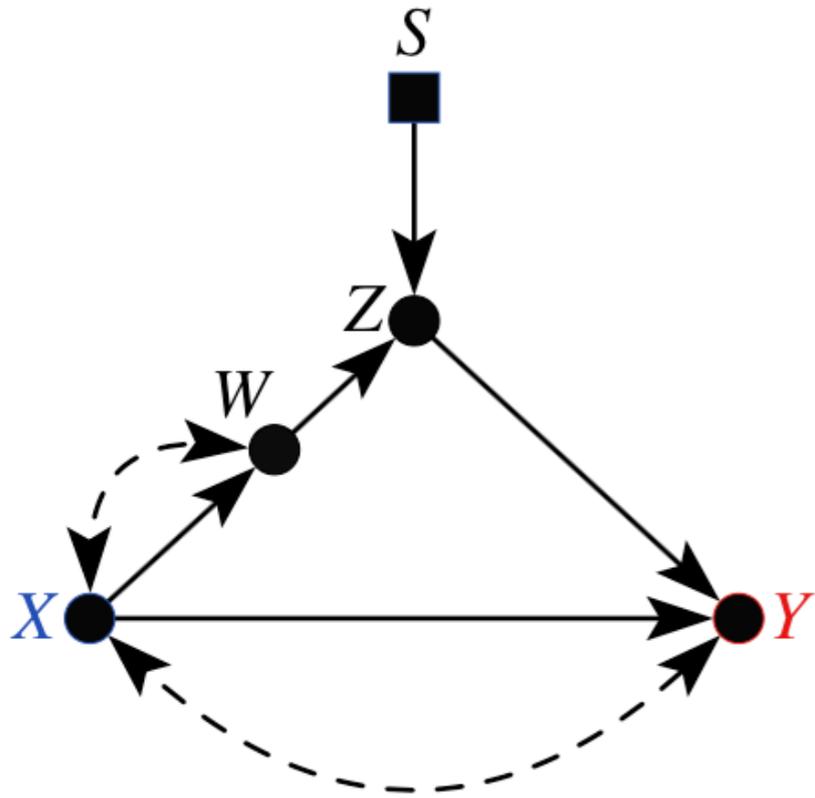
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Finding invariances: beyond direct transportability

A more elaborate example:

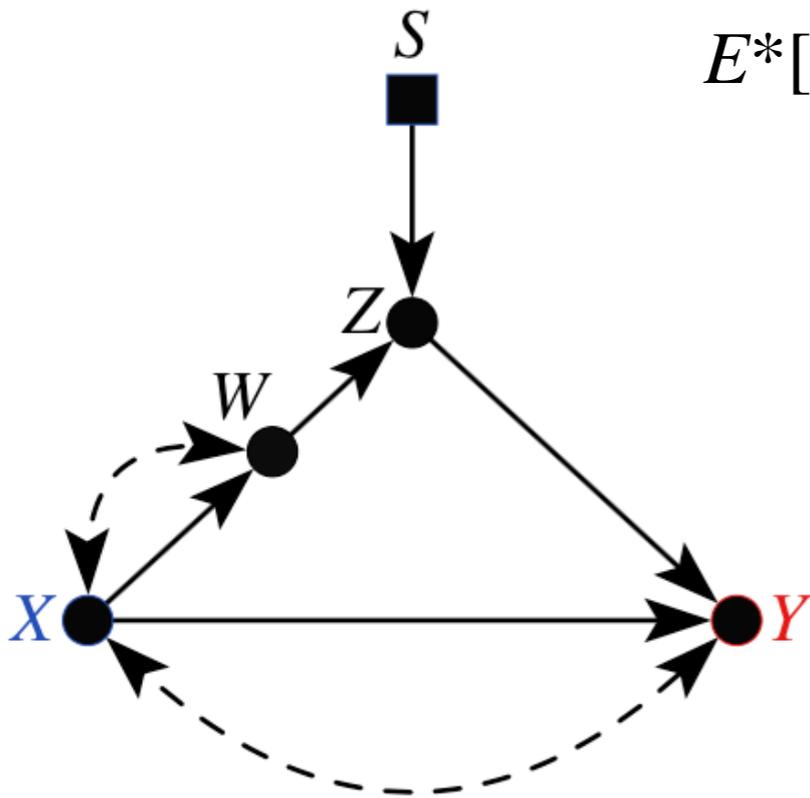
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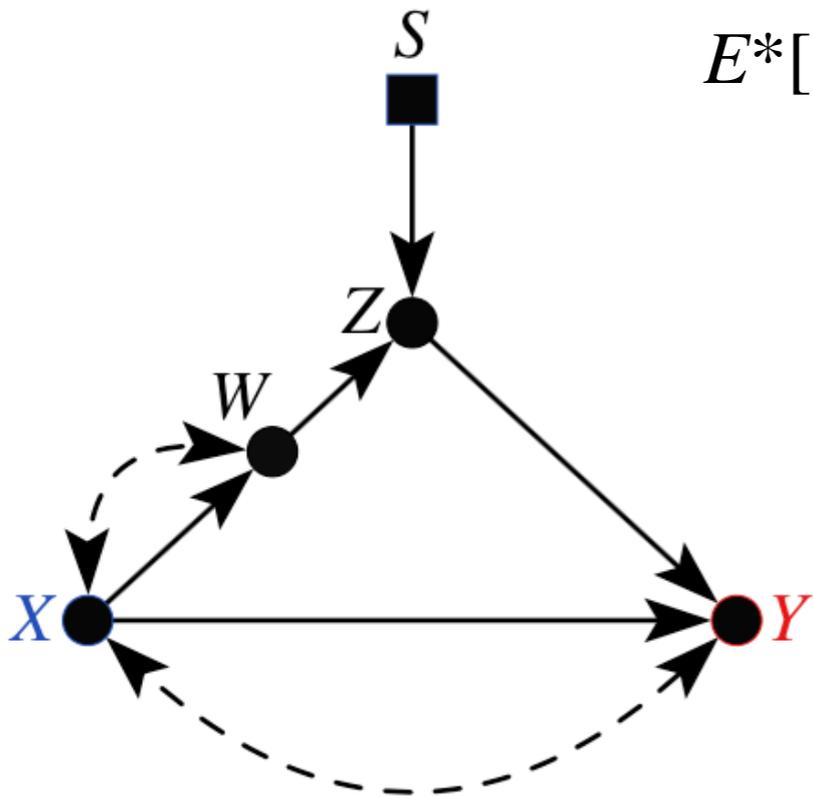
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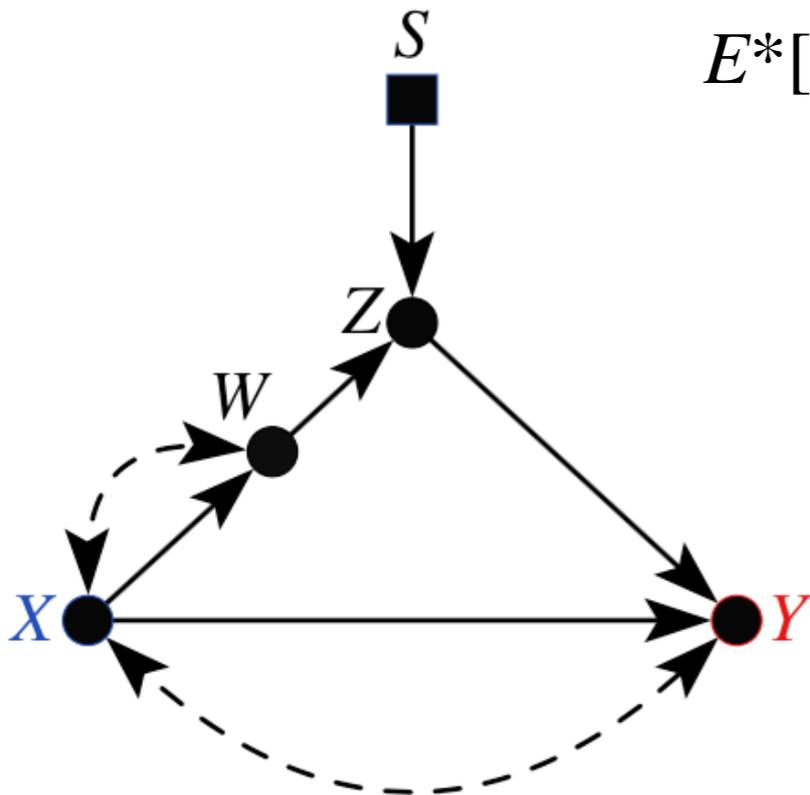
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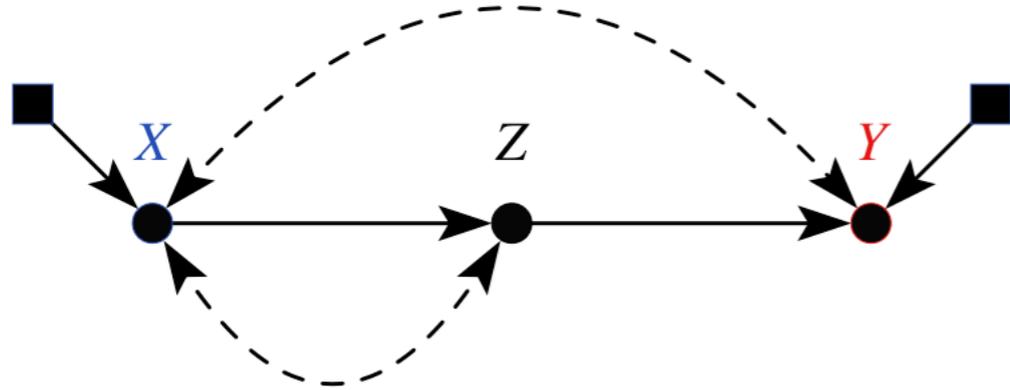
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Now let us extend to **multiple populations**, each with **different experimental conditions**: for instance, in one domain only X was randomized while in another domain only Z was randomized... and so on.

Multiple Populations

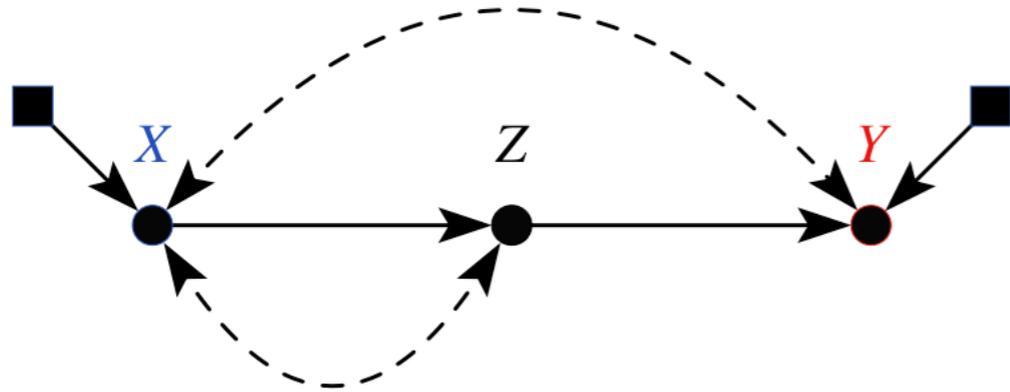
Multiple Populations

Π^A :



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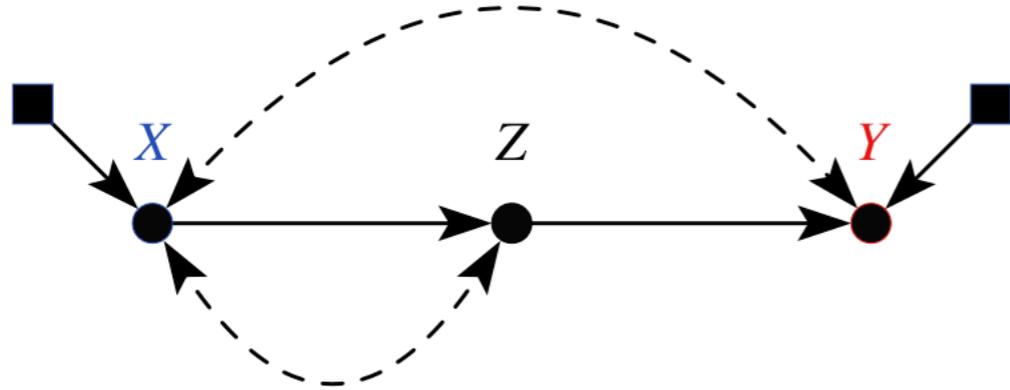
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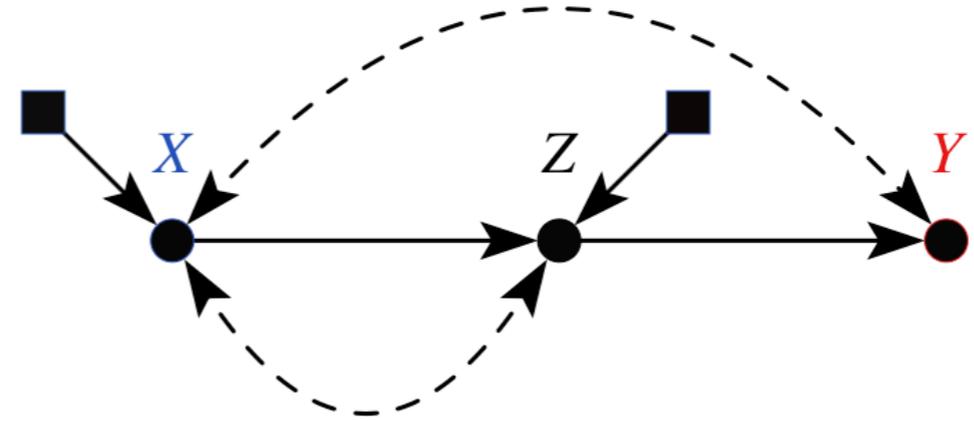
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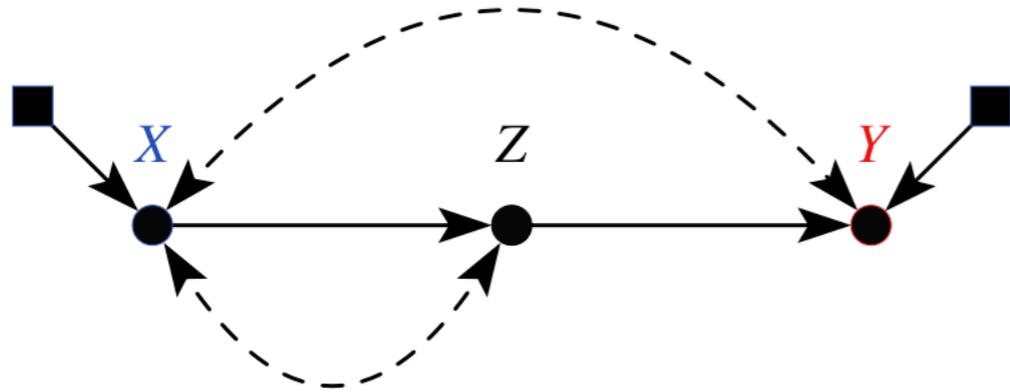
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Π^B :



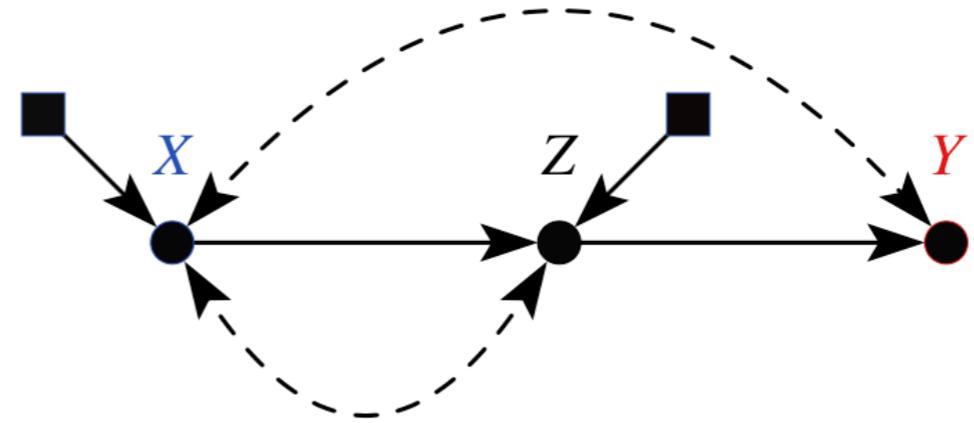
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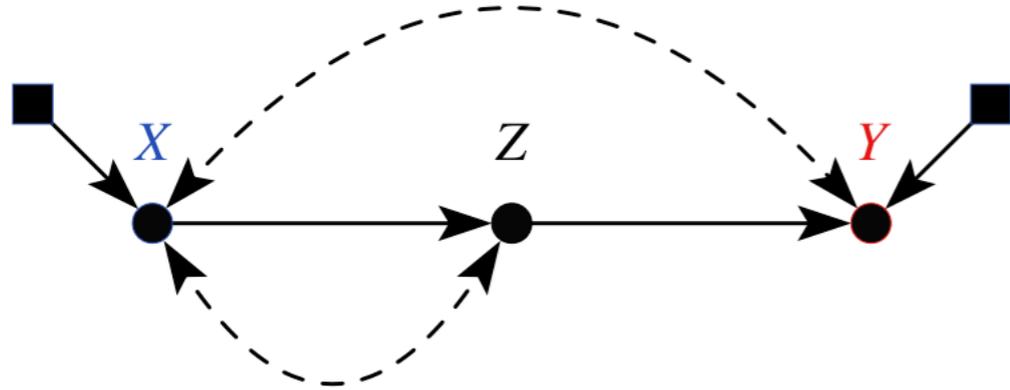
Π^B :



Not transportable from B.

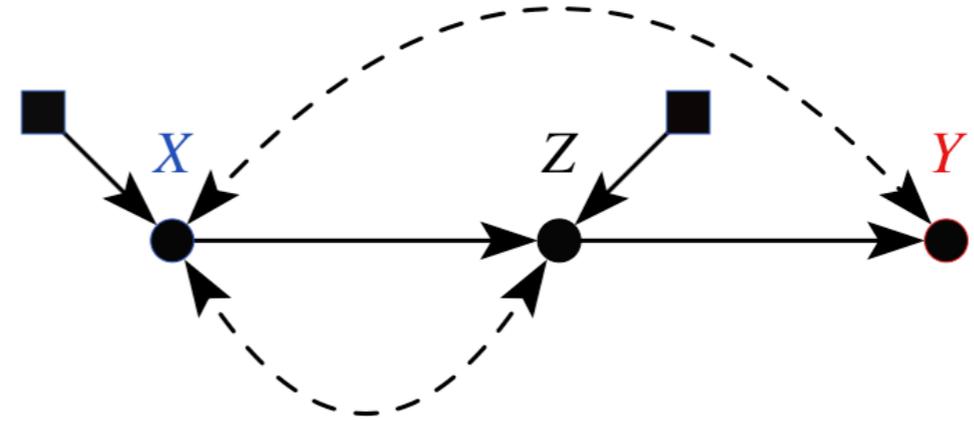
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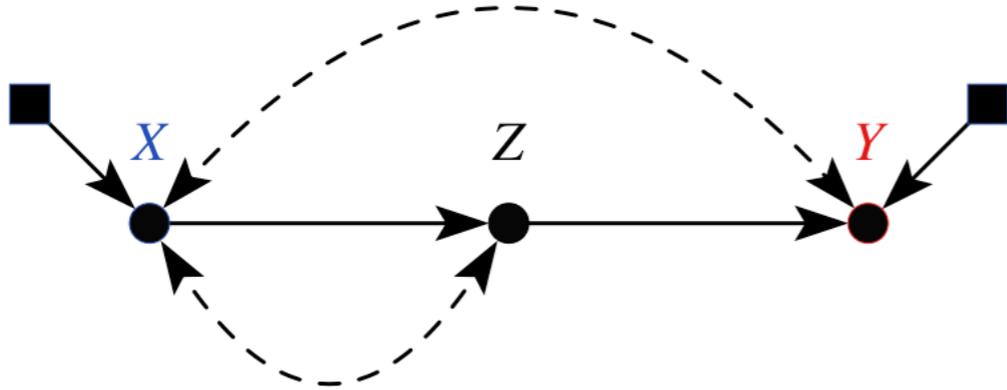


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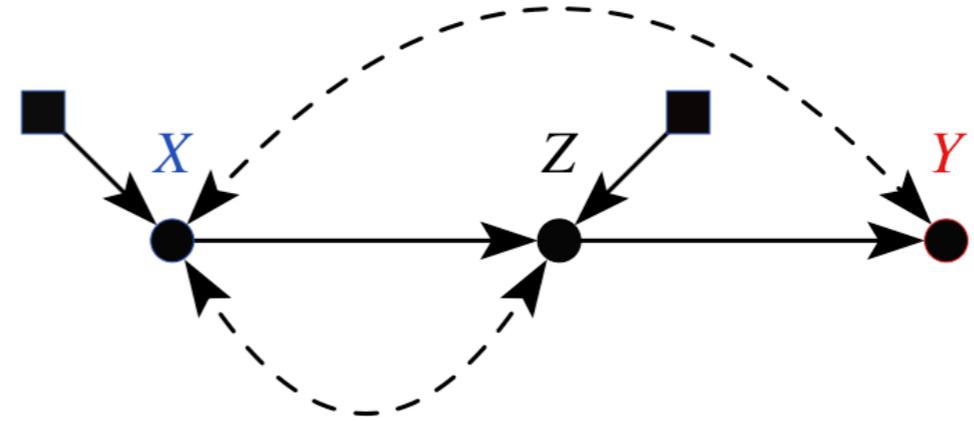
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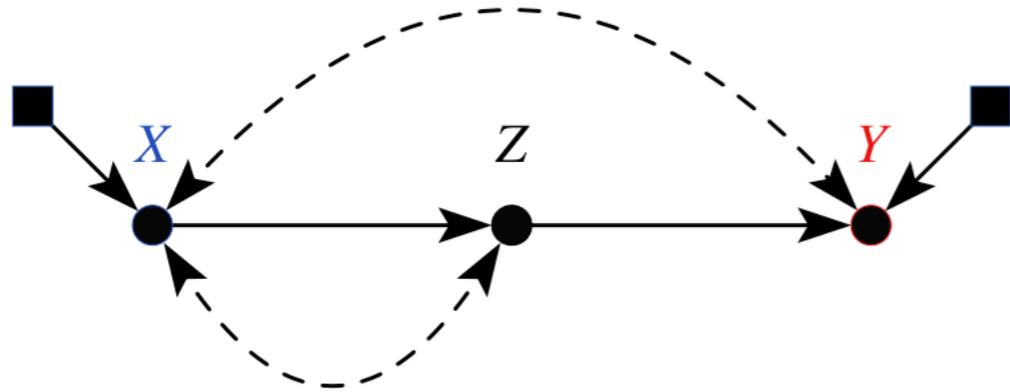
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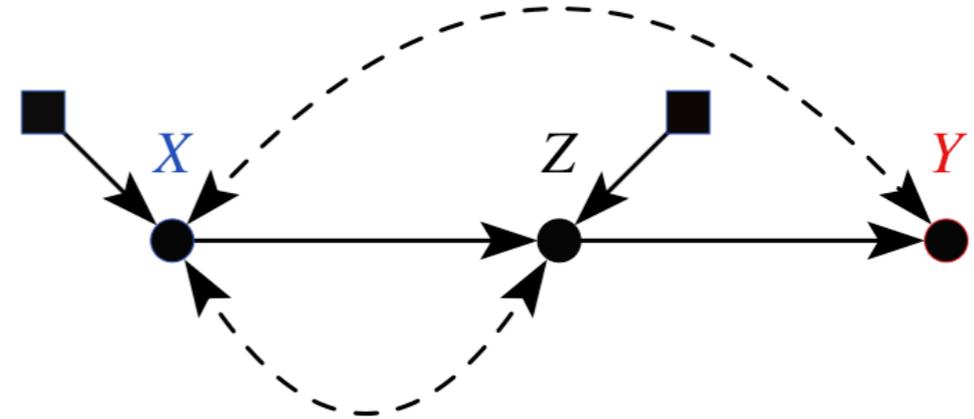
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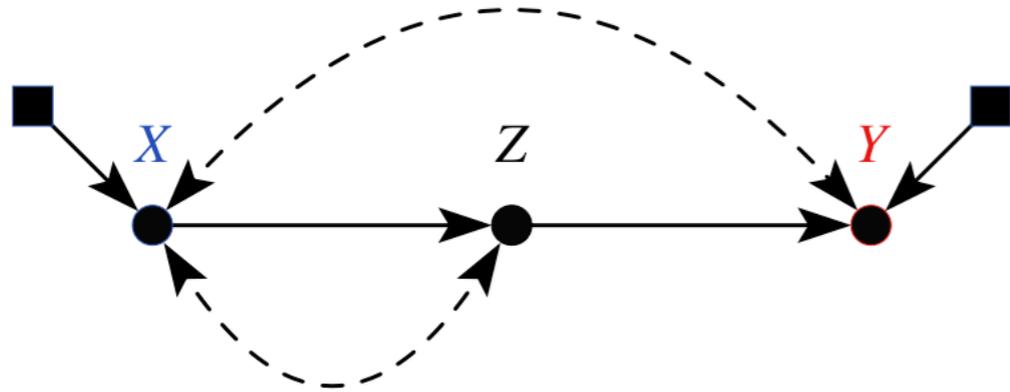
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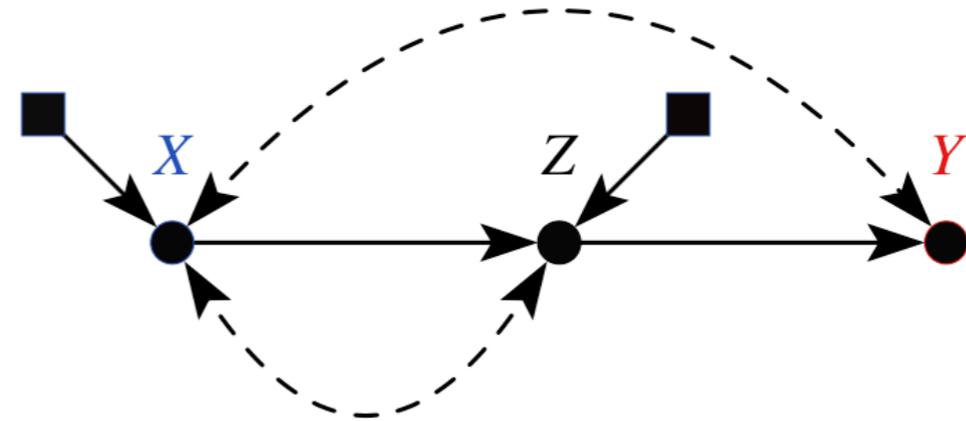
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You do not need to derive each case by hand.

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We have a ***complete algorithms*** that can decide how to combine results of several experimental and observational studies, each conducted on a different population and under a different set of conditions, so as to construct a valid estimate of the effect size for the target population.

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FUSION DEMO 1

Selection Bias

Selected Subpopulation → General Population

Selection bias vs Confounding bias

Warning: some economists use "selection bias" to denote confounding bias

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More generally, we use selection bias to mean bias due to ***preferential selection of units into the study sample.***

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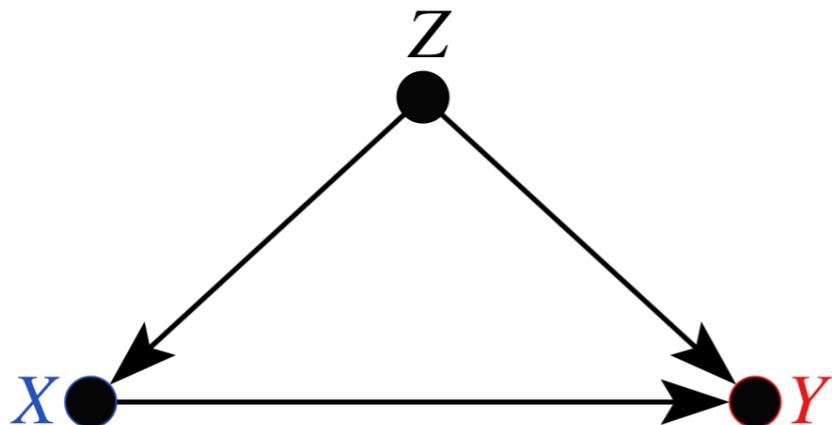
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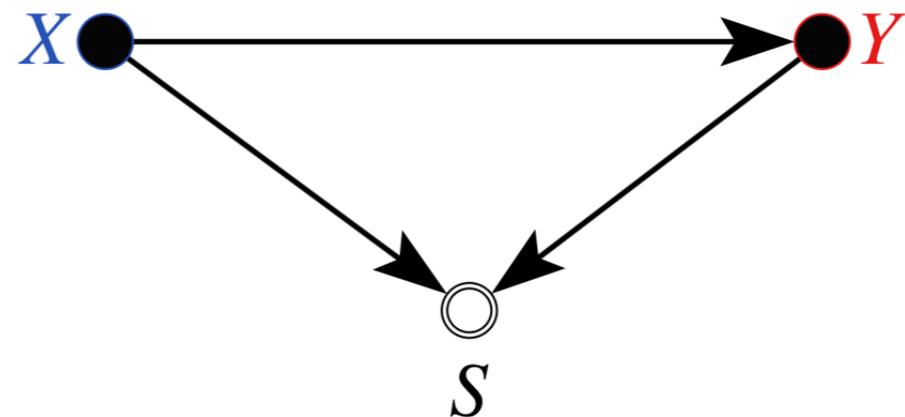
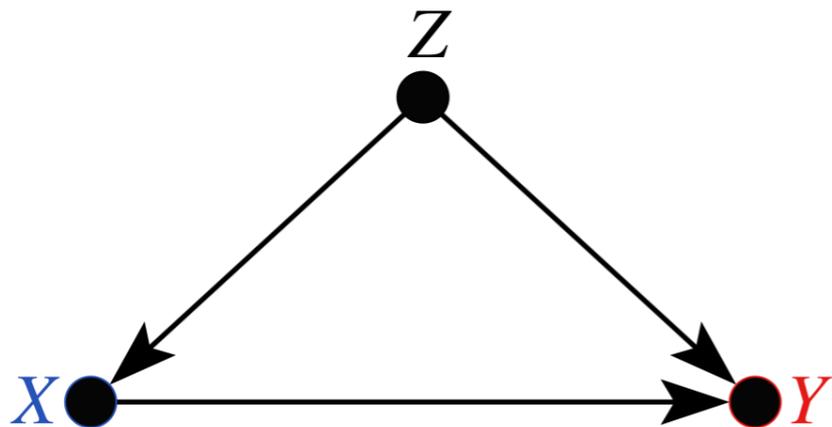
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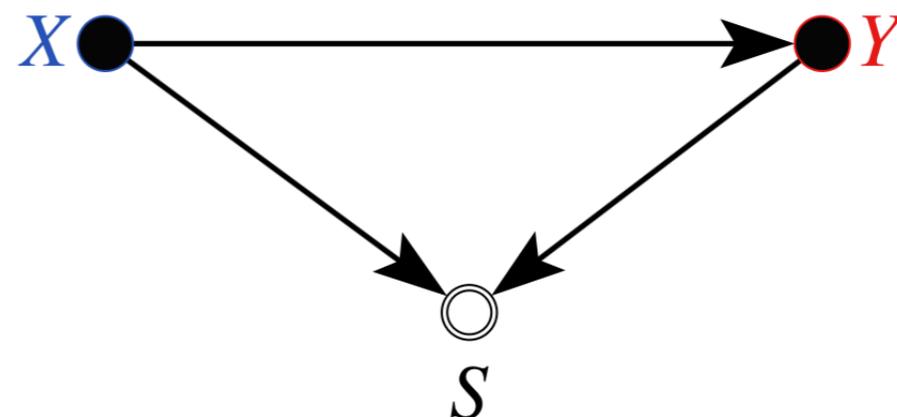
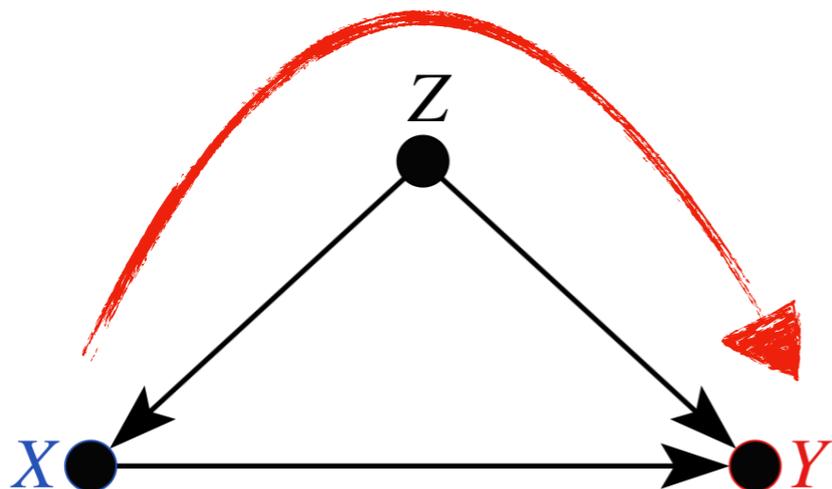
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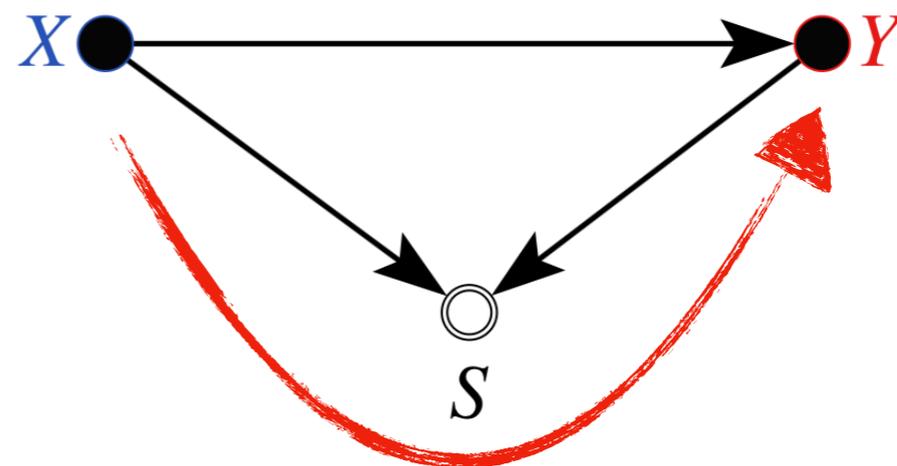
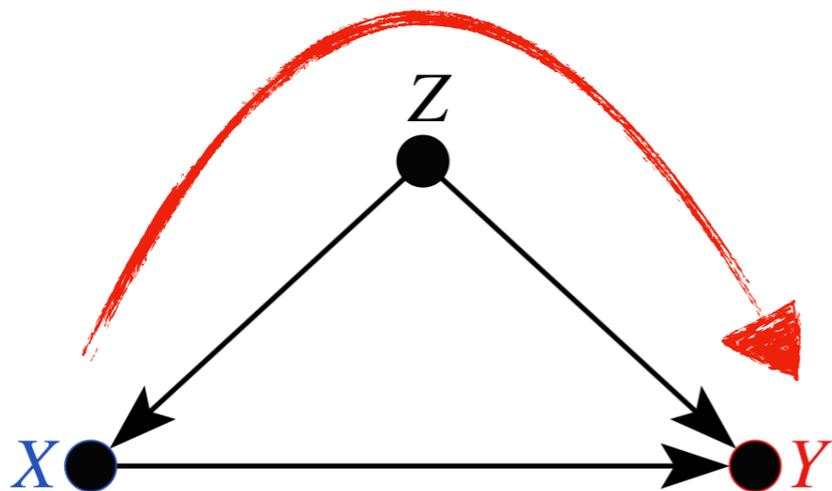
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Formalizing selection bias

1) *What do we want to know? (Query)*

- *Conditional expectation on general population: $P(y|x)$*
- *Causal effect on general population: $E[Y|do(x)]$*

Formalizing selection bias

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- Causal effect on general population: $E[Y|do(x)]$

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- Observational/Experimental data in the study sample ($S = 1$). May or may not have census data for some variables Z in the general population.

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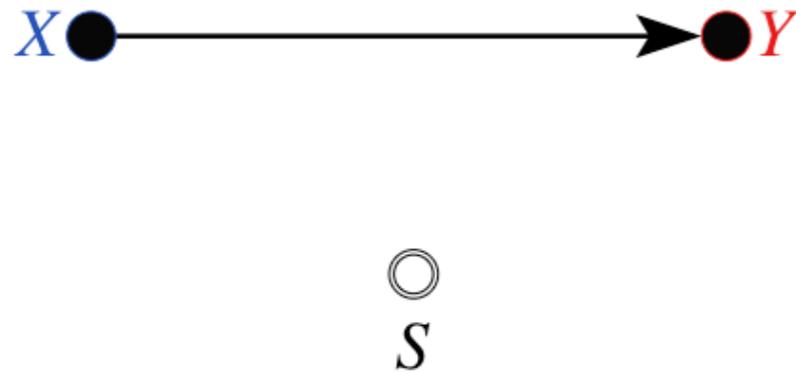
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- Here: **nonparametric, qualitative description** of the determinants of inclusion of units in the study sample.

Encoding the selection mechanism

Again we extend our causal diagram with “selection nodes” (S) which now indicate ***selection to the study sample ($S = 1$), or not ($S = 0$)***. Our target of inference is a quantity on the population as a whole, ***not conditioning on S*** .

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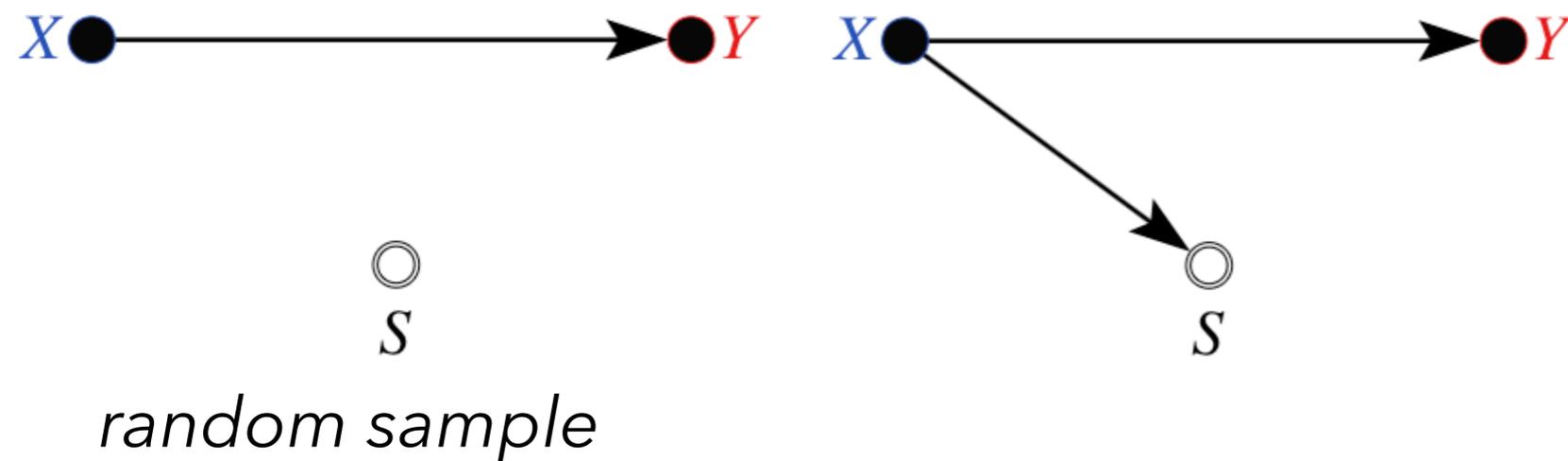


○
 S

random sample

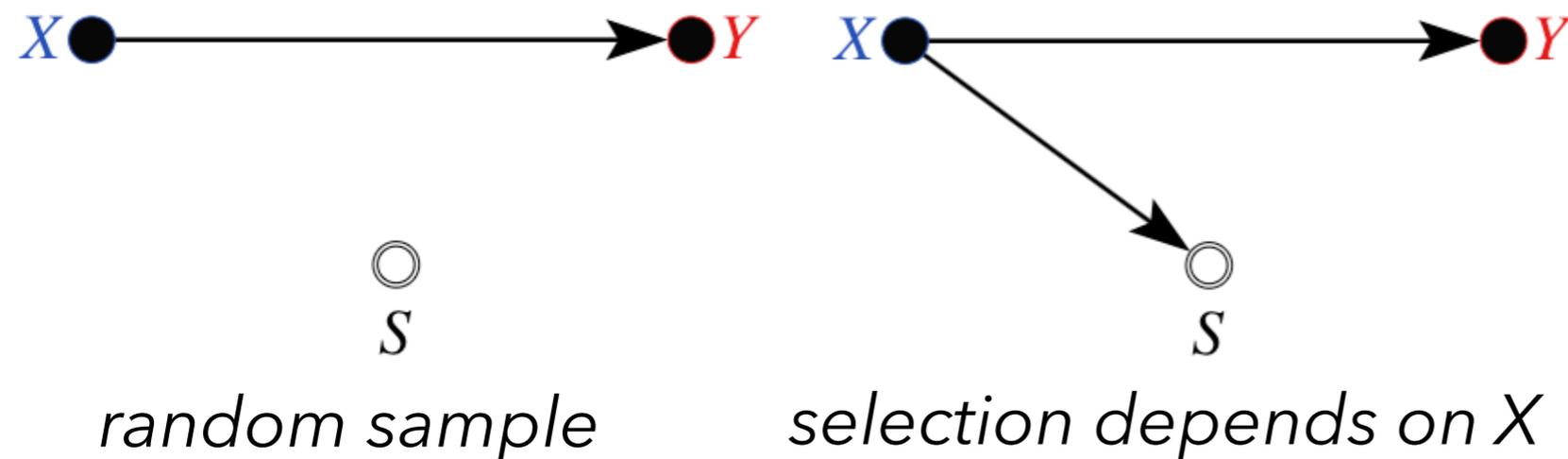
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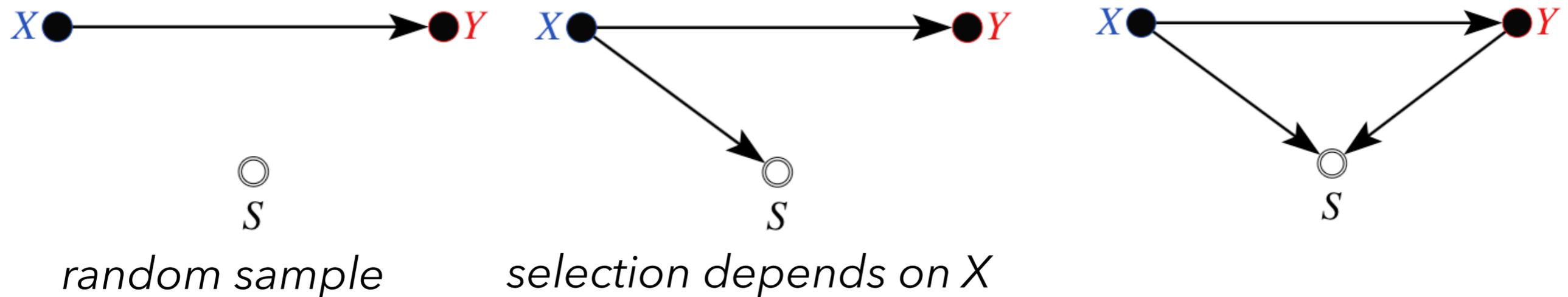
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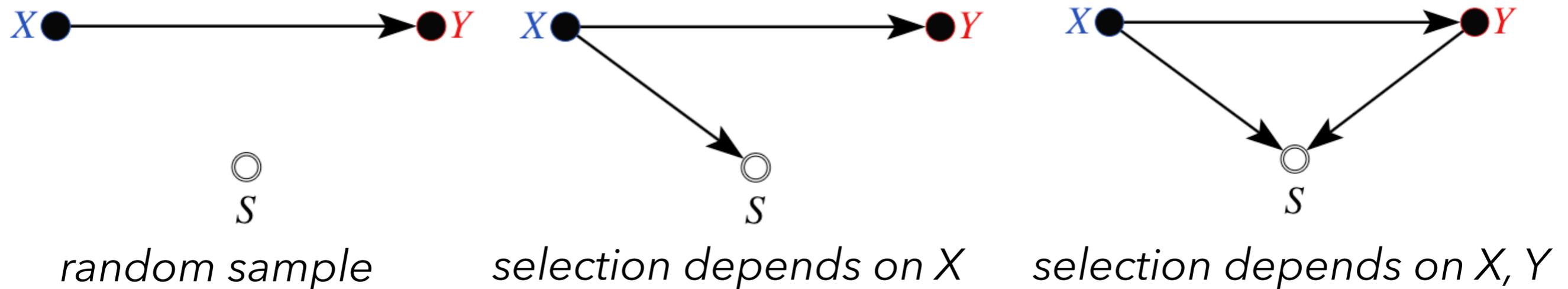
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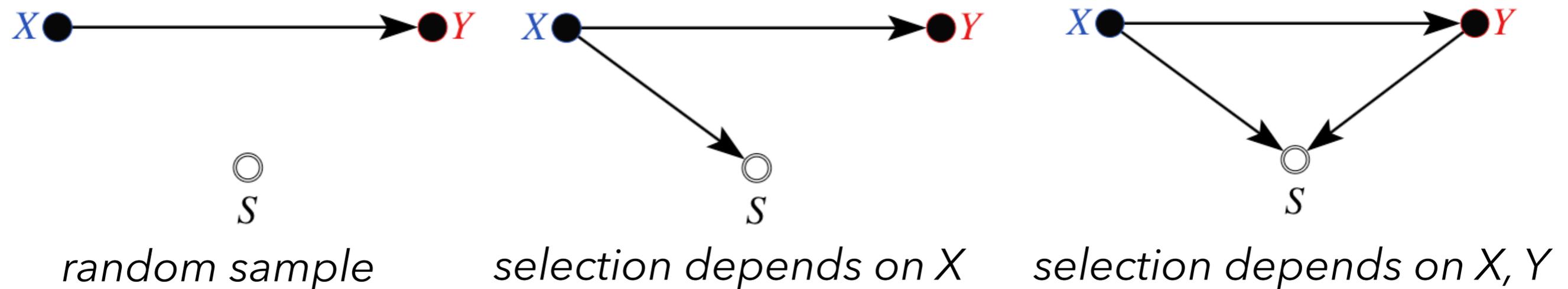
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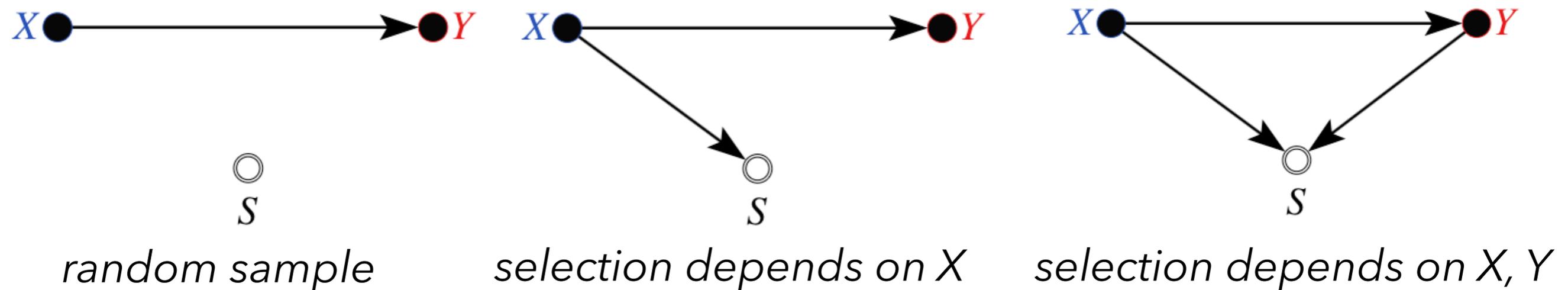
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Symbolically, our task is to express the query in terms of the available data, that is, the **distribution under selection bias** $P(V | S = 1)$ – or more concisely $P(V | s)$ – and the **census data** we have available (if any).

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Graphically, we will check for separation of the selection mechanism S from key variables of interest that compose our query.

Recovering conditional distributions from selection

Very simple necessary and sufficient condition for conditional distributions.

Recovering conditional distributions from selection

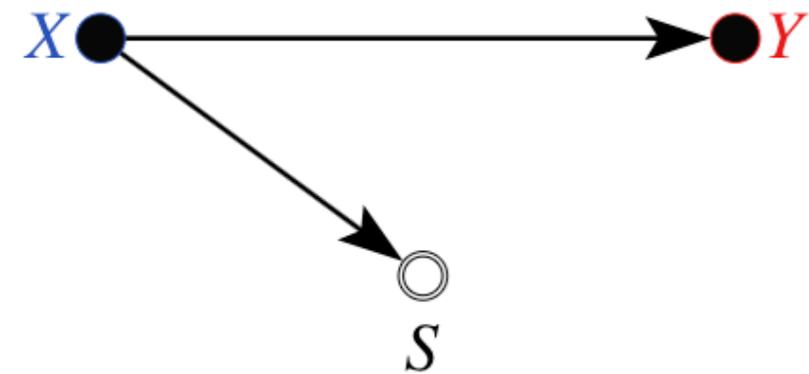
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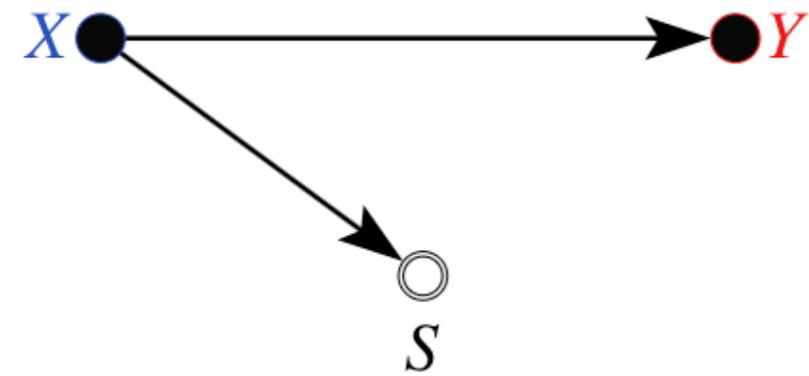
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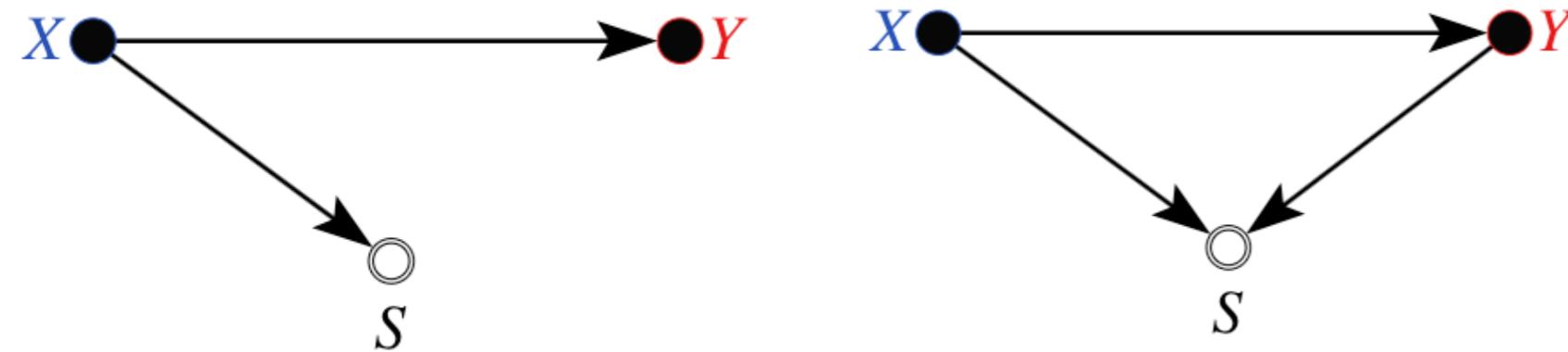


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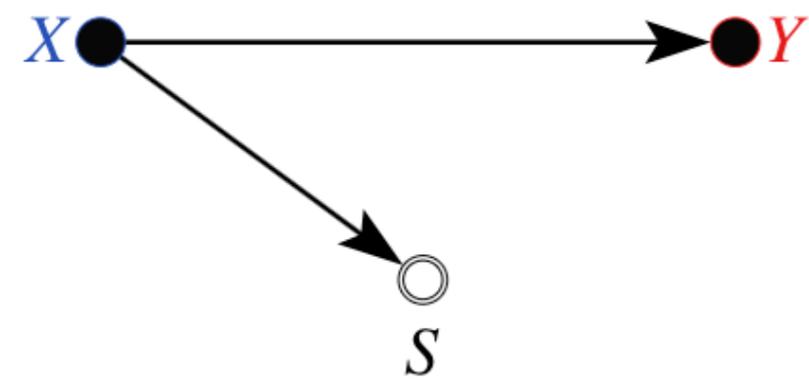


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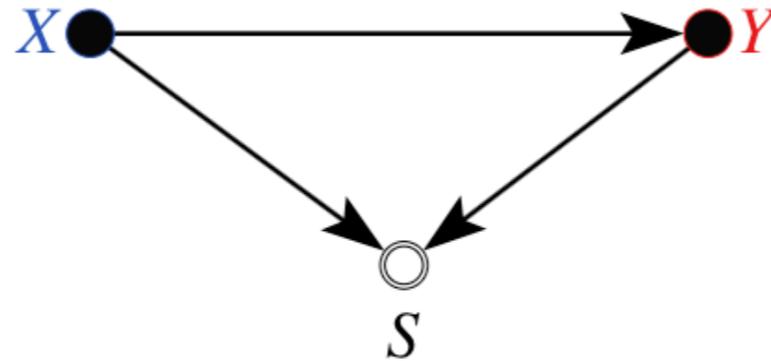
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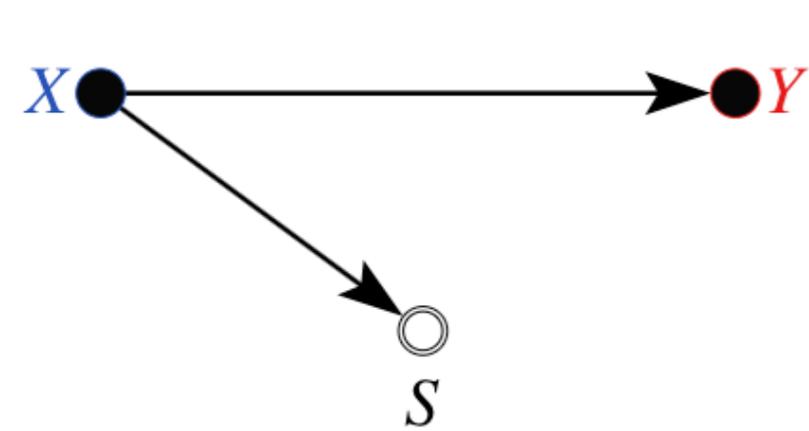


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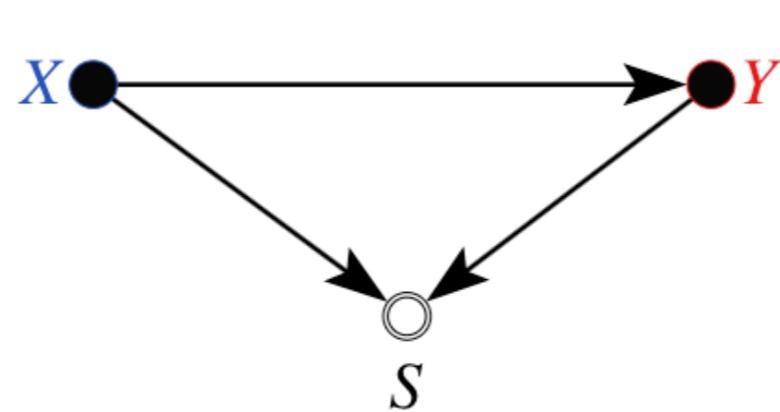
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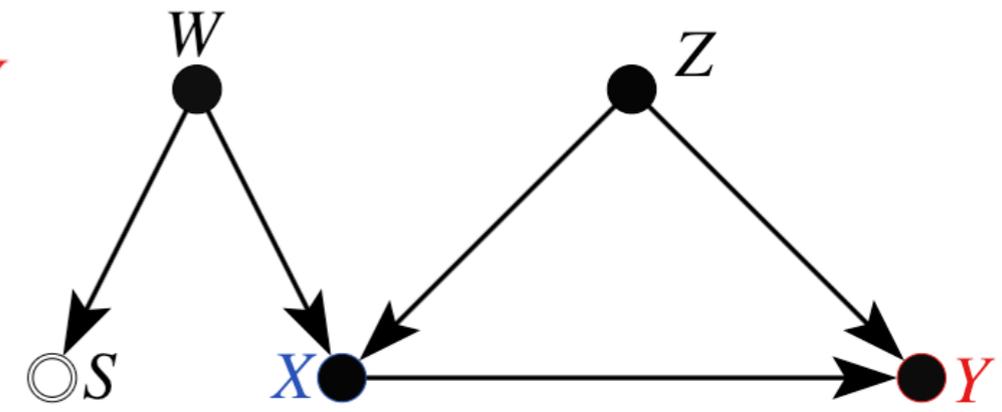
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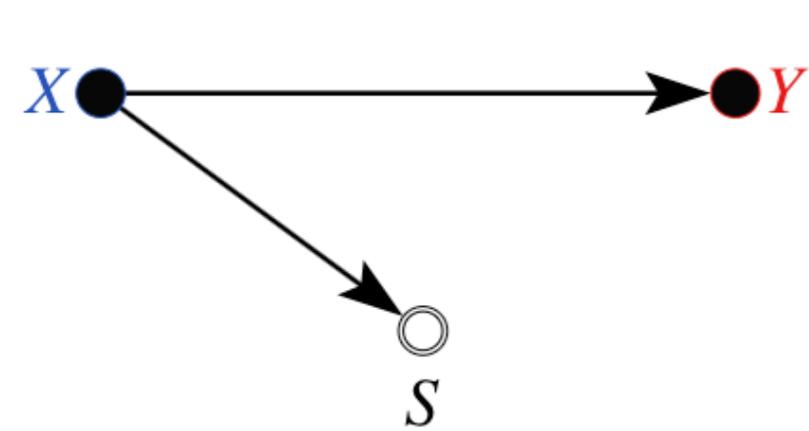
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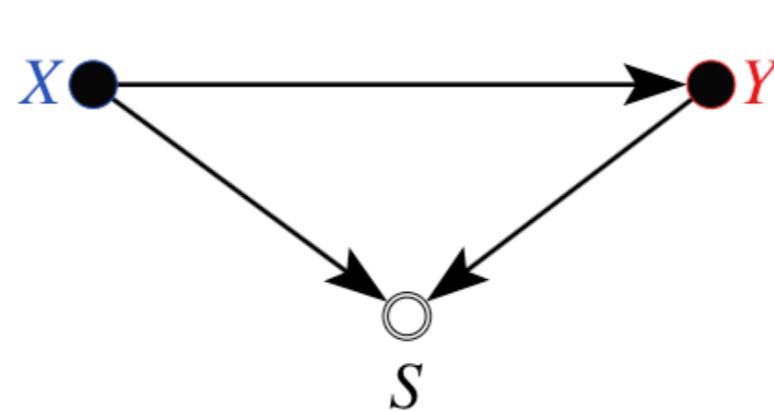
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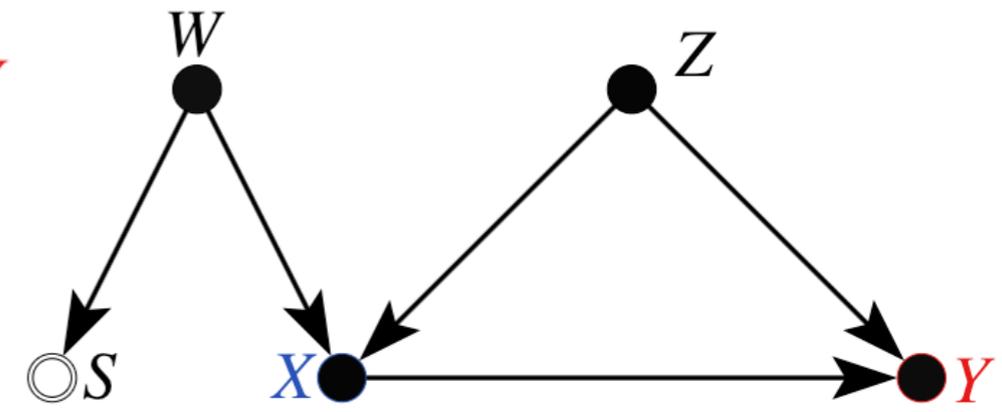
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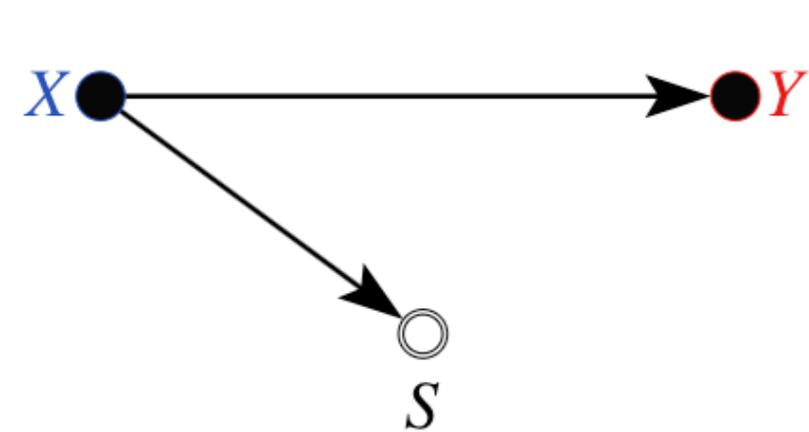


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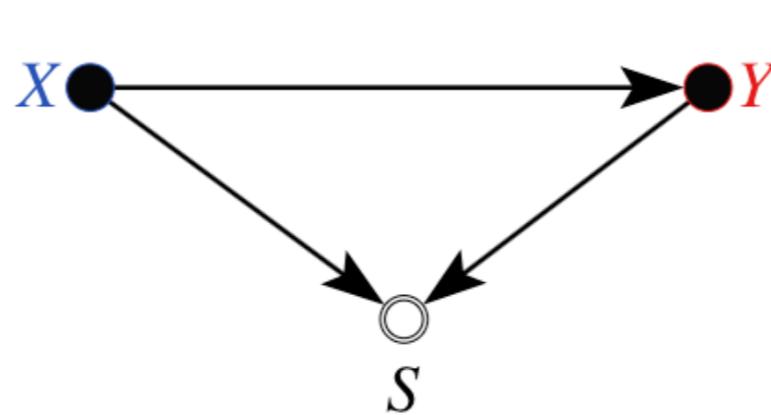
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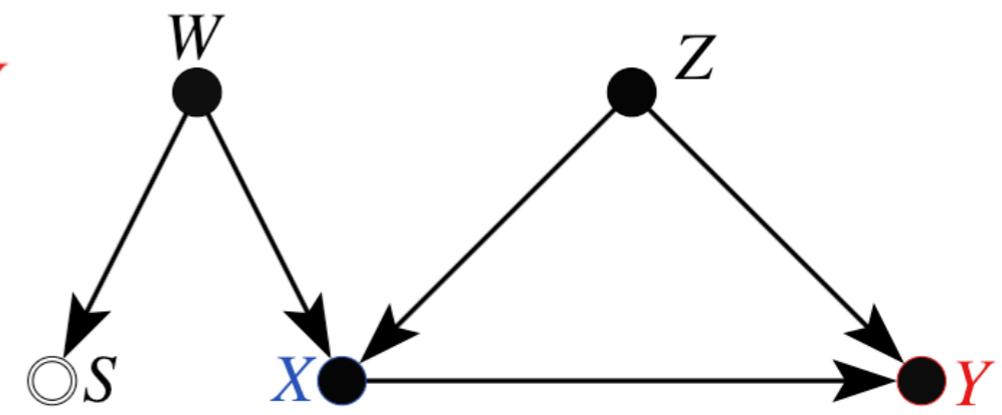
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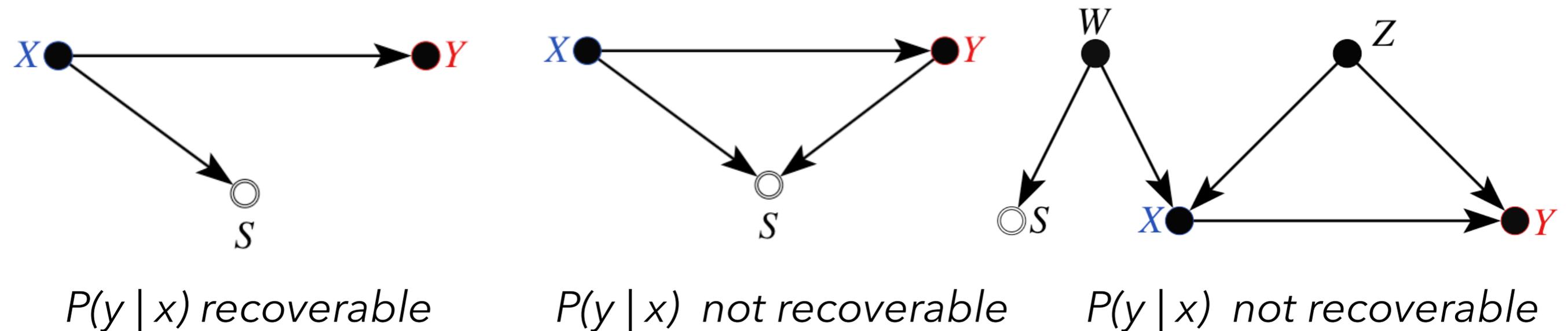
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Note this is different from recovering the causal effect $P(y | do(x))$.

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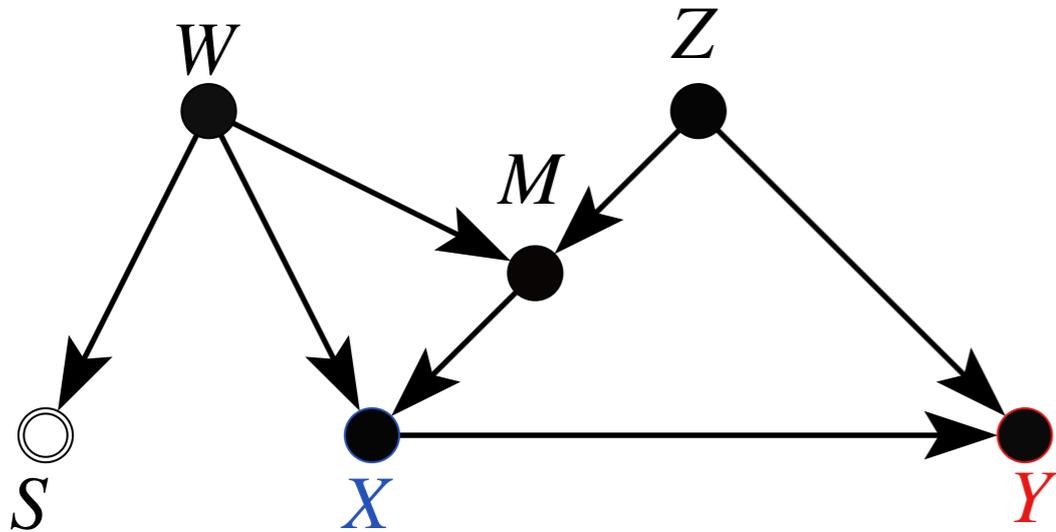


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For instance, in the third model, $P(y|x)$ is not recoverable, while $P(y|\text{do}(x))$ is, as we show next.

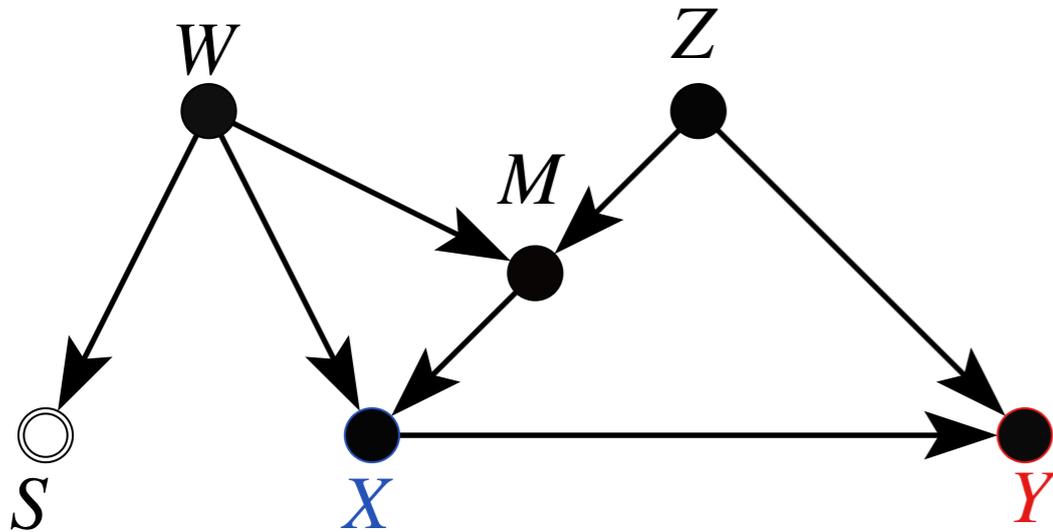
Recovering causal effects from selection and confounding

Do we need external data?



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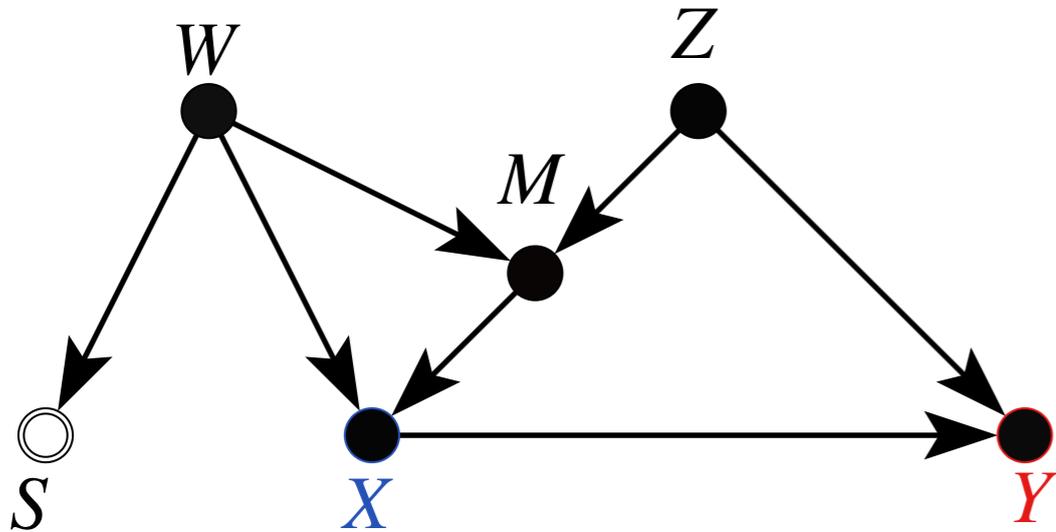
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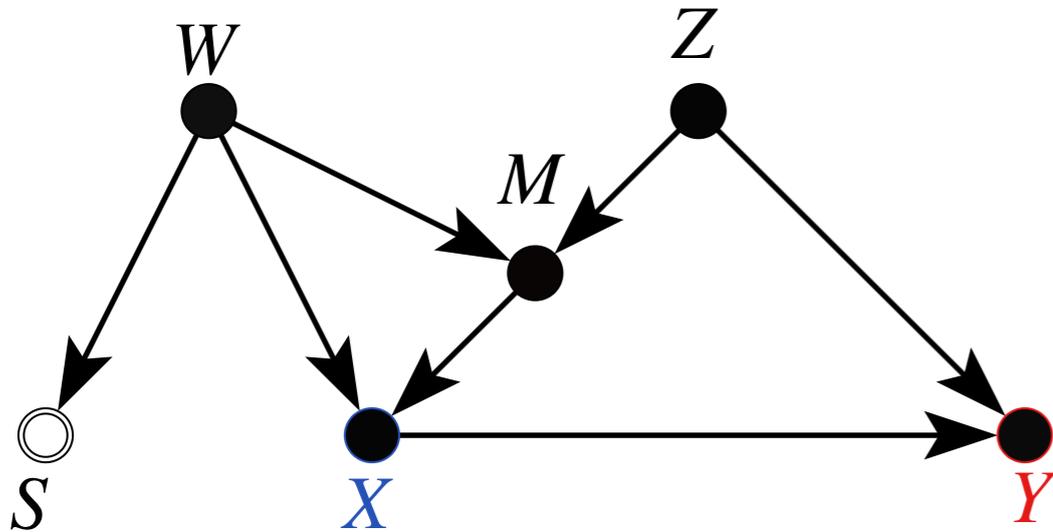
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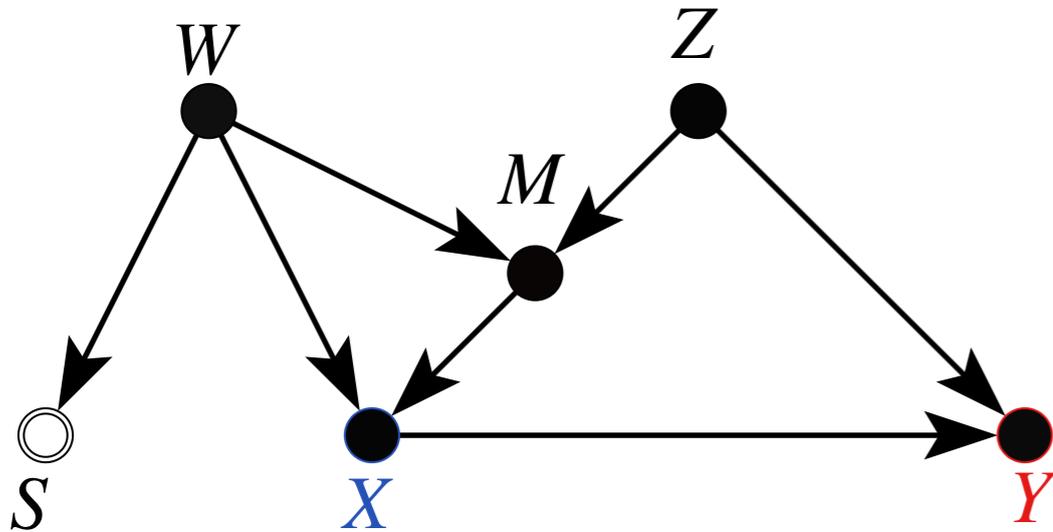


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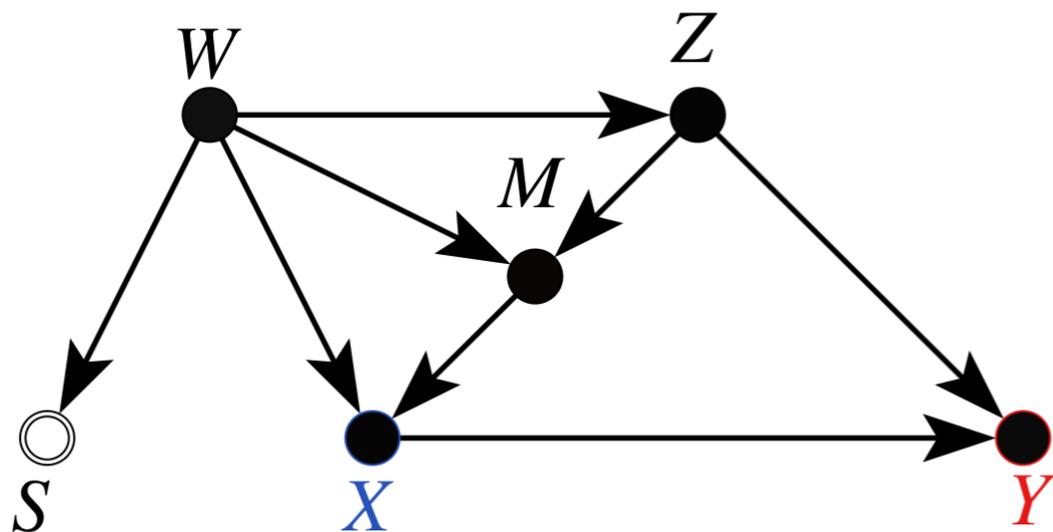
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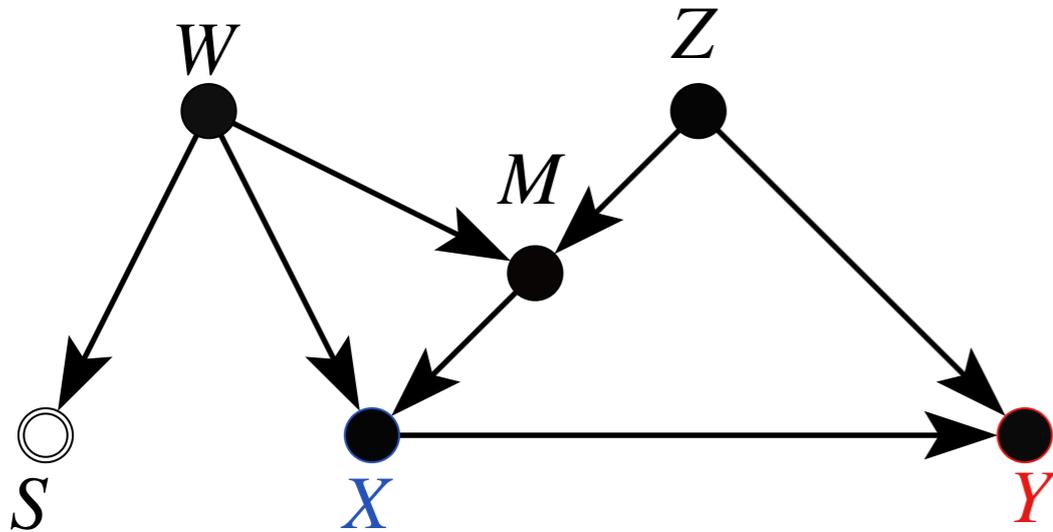
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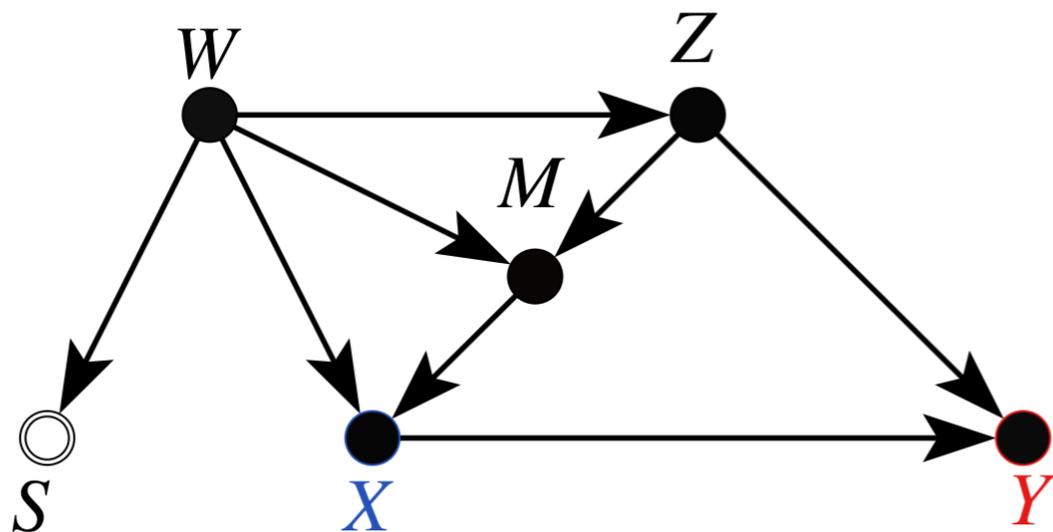


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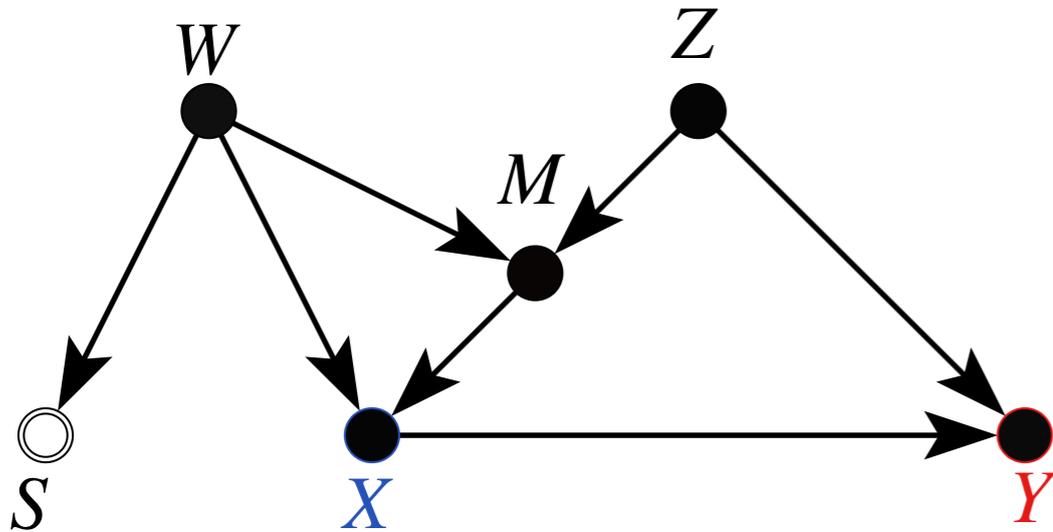
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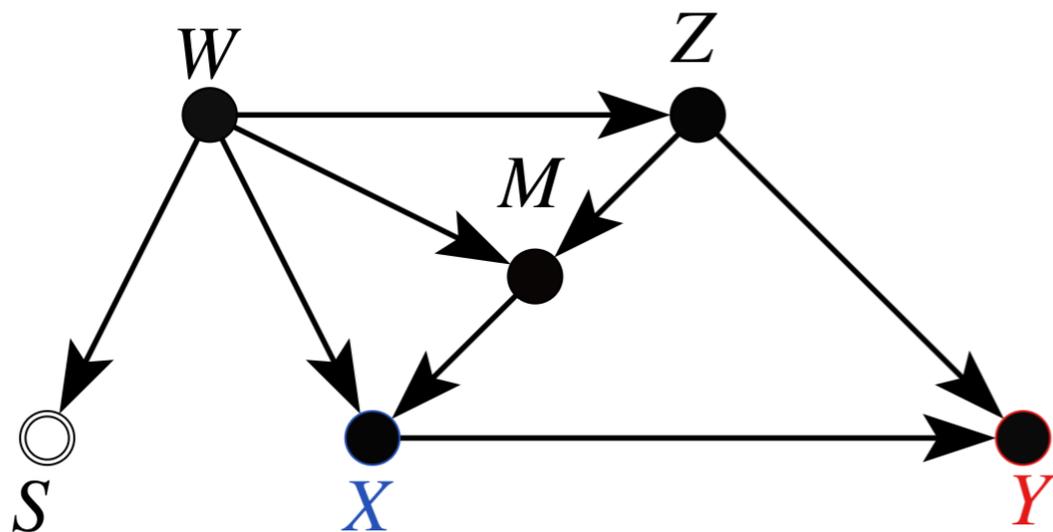


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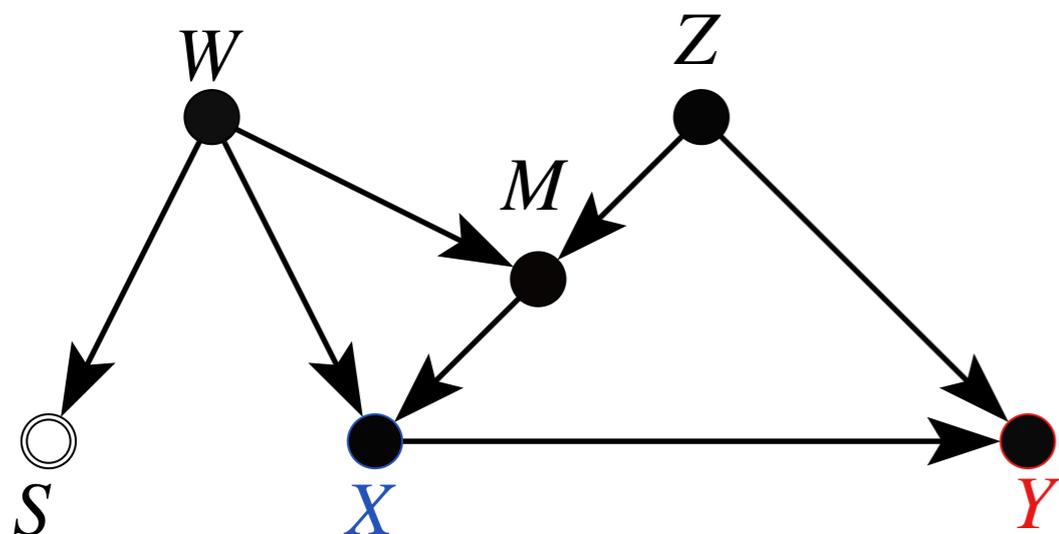


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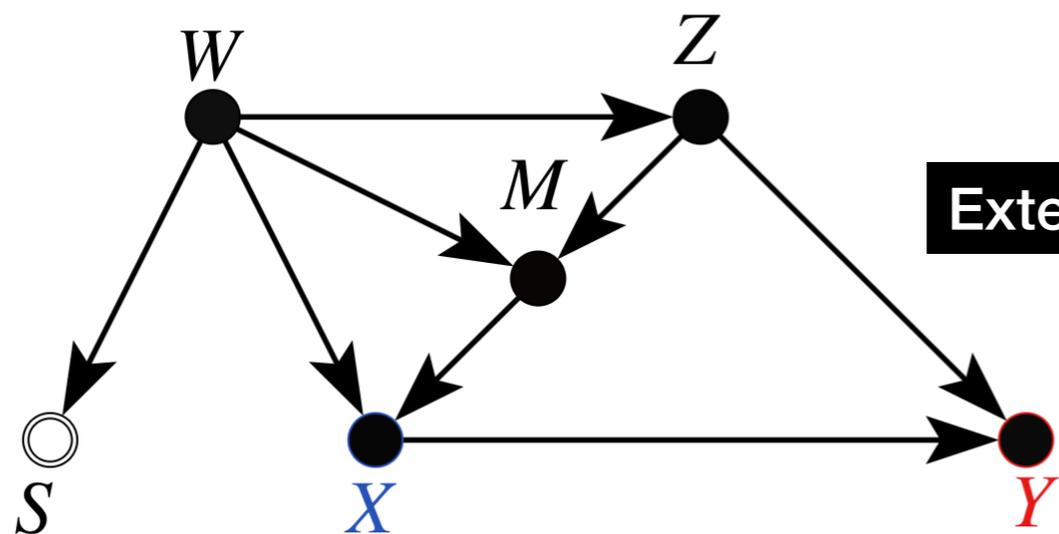


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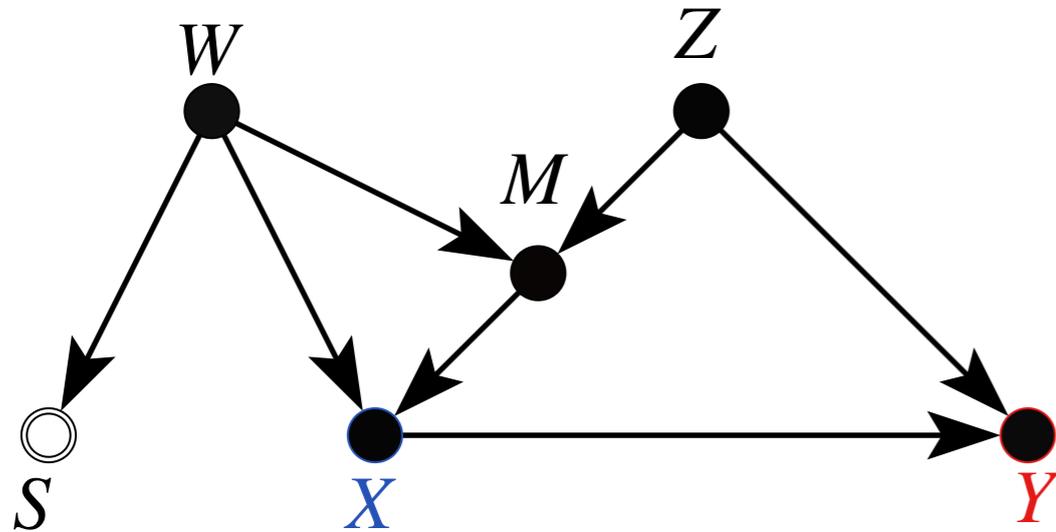
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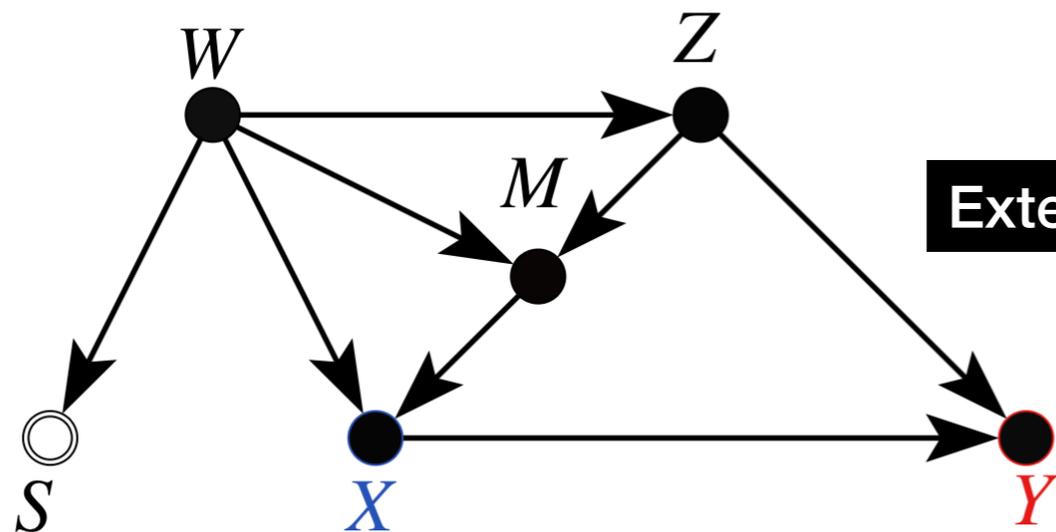


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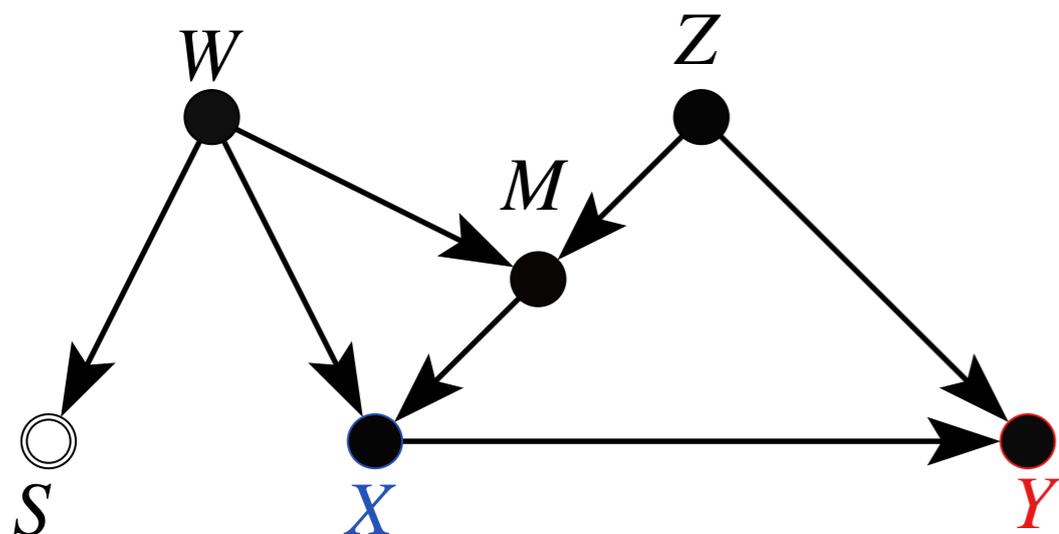
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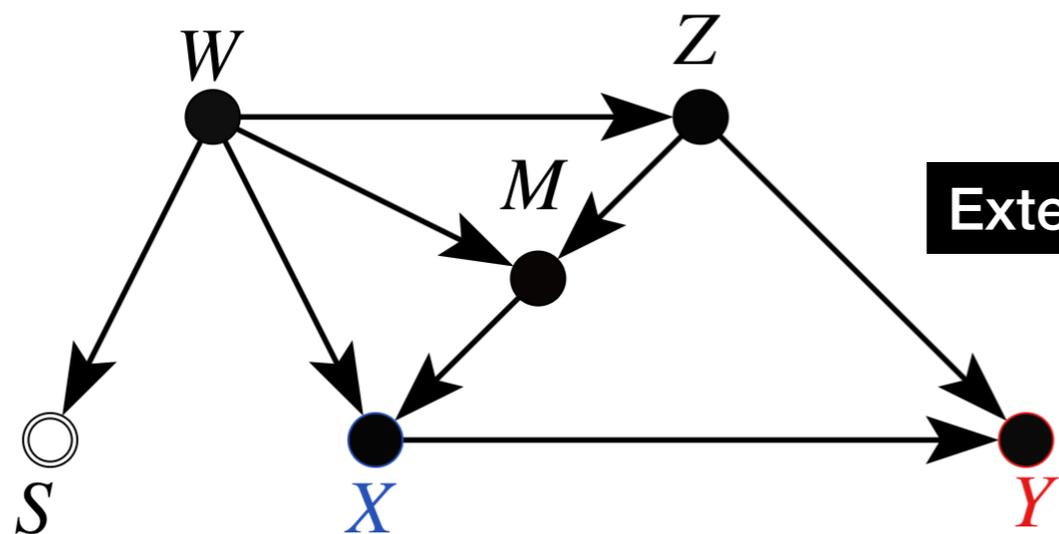


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Or external data on W!

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***Recovery without external data:** we have complete algorithms for recovering from selection and confounding biases, both for markovian and semi-markovian models.*

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FUSION DEMO 2

Data Fusion

$(d_1, d_2, d_3, d_4) \rightarrow (d'_1, d'_2, d'_3, d'_4)$

Putting it all together

We can describe each data collection as the tuple:

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		Dataset 1	Dataset 2	Dataset 3
d_1	Population	Los Angeles	New York	Texas
d_2	Obs. / Exp.	Experimental	Observational	Experimental
	Treat.Assign.	Randomized Z_1	-	Randomized Z_2
d_3	Sampling	Selection on Age	Selection on SES	-
d_4	Measured	X_1, Z_1, W, M, Y_1	X_1, X_2, Z_1, N, Y_2	X_2, Z_1, W, L, M, Y_1

Putting it all together

We can describe each data collection as the tuple:

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		Dataset 1	Dataset 2	Dataset 3
d ₁	Population	Los Angeles	New York	Texas
d ₂	Obs. / Exp.	Experimental	Observational	Experimental
	Treat.Assign.	Randomized Z ₁	-	Randomized Z ₂
d ₃	Sampling	Selection on Age	Selection on SES	-
d ₄	Measured	X ₁ , Z ₁ , W, M, Y ₁	X ₁ , X ₂ , Z ₁ , N, Y ₂	X ₂ , Z ₁ , W, L, M, Y ₁

-Observational Causal Inference: $(d_1, \text{see}(x), d_3, d_4) \rightarrow (d_1, \text{do}(x), d_3, d_4)$

Putting it all together

We can describe each data collection as the tuple:

$(d_1, d_2, d_3, d_4) = (\text{population, obs/exp., sampling selection, observed data})$

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d_1	Population	Los Angeles	New York	Texas
d_2	Obs. / Exp.	Experimental	Observational	Experimental
	Treat.Assign.	Randomized Z_1	-	Randomized Z_2
d_3	Sampling	Selection on Age	Selection on SES	-
d_4	Measured	X_1, Z_1, W, M, Y_1	X_1, X_2, Z_1, N, Y_2	X_2, Z_1, W, L, M, Y_1

-Observational Causal Inference: $(d_1, \text{see}(x), d_3, d_4) \rightarrow (d_1, \text{do}(x), d_3, d_4)$

- Sampling Selection Bias: $(d_1, d_2, \text{select}(\text{age}), d_4) \rightarrow (d_1, d_2, \{ \}, d_4)$

- Transportability: $(LA, d_2, d_3, d_4) \rightarrow (NY, d_2, d_3, d_4)$

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		Dataset 1	Dataset 2	Dataset 3
d ₁	Population	Los Angeles	New York	Texas
d ₂	Obs. / Exp.	Experimental	Observational	Experimental
	Treat.Assign.	Randomized Z ₁	-	Randomized Z ₂
d ₃	Sampling	Selection on Age	Selection on SES	-
d ₄	Measured	X ₁ , Z ₁ , W, M, Y ₁	X ₁ , X ₂ , Z ₁ , N, Y ₂	X ₂ , Z ₁ , W, L, M, Y ₁

-Observation

- Sampling Selection

- Transportability

In general: $(d_1, d_2, d_3, d_4) \rightarrow (d'_1, d'_2, d'_3, d'_4)$

$do(x), d_3, d_4)$

$(d_1, d_2, \{ \}, d_4)$

$, d_3, d_4)$

Conclusions

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Software under development: Causal Fusion.

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Thank you!

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